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DID THE UNIVERSE RECOMBINE? NEW SPECTRAL CONSTRAINTS ON REHEATING †

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ABSTRACT

One still cannot conclusively assert that the universe underwent a neutral phase, despite the new COBE FIRAS limit $y < 2.5\times 10^{-5}$ on Compton y-distortions of the cosmic microwave background. Although scenarios where the very early ($z\sim 1,000$) ionization is thermal, caused by IGM temperatures exceeding 10^4K, are clearly ruled out, there is a significant loophole for cosmologies with typical CDM parameters if the dominant ionization mechanism is photoionization. If the ionizing radiation has a typical quasar spectrum, then the y-constraint implies roughly $h^{3/2}\Omega_b\Omega_0^{-1/4} < 0.1$ for fully ionized models. This means that BDM models with $\Omega_0\approx 0.15$ and reionization at $z\approx 1,000$ are strongly constrained even in this very conservative case, and can survive the y test only if most of the baryons form BDM around the reionization epoch.

† Revised version

1. INTRODUCTION

Recombination of the primeval plasma is commonly assumed but was by no means inevitable. Theories exist that predict early reionization are as diverse as those invoking primordial seed fluctuations that underwent early collapse and generated sources of ionizing radiation, and models involving decaying or annihilating particles. The former class includes cosmic strings and textures, as well as primordial isocurvature baryon fluctuations. The latter category includes baryon symmetric cosmologies as well as decaying exotic particles or neutrinos.

The Compton y-distortion of the cosmic microwave background (CMB) provides a unique constraint on the epoch of reionization. In view of the extremely sensitive recent FIRAS limit of $y < 2.5\times10^{-5}$, we have reinvestigated constraints on the early ionization history of the intergalactic medium (IGM), and have chosen to focus on what we regard as the most important of the non-standard recombination history models, namely the primordial isocurvature baryon scenario involving a universe dominated by baryonic dark matter (BDM), as advocated by Peebles (1987), Gnedin & Ostriker (1992) (hereafter "GO"), Cen, Ostriker & Peebles (1993) and others. This class of models takes the simplest matter content for the universe, namely baryons, to constitute dark matter in an amount that is directly observed and is even within the bounds of primordial nucleosynthesis, if interpreted liberally, and can reconstruct essentially all of the observed phenomena that constrain large-scale structure. The BDM model is a non-starter unless the IGM underwent very early reionization, in order to avoid producing excessive CMB fluctuations on degree scales. Fortunately, early nonlinearity is inevitable with BDM initial conditions, $\delta\rho/\rho \propto M^{-5/12}$, corresponding to a power-spectrum $\langle\delta_k^2\rangle \propto k^{-1/2}$ for the observationally preferred choice of spectral index (Cen, Ostriker & Peebles 1993).

Is it possible that the IGM has been highly ionized since close to the standard recombination epoch at $z \approx 1100$? Perhaps the most carefully studied BDM scenario in which this happens is that by GO. In their scenario, $\Omega_0 = \Omega_{b0} \approx 0.15$. Shortly after recombination, a large fraction of the mass condenses into faint stars or massive black holes, releasing energy that reionizes the universe and heats it to $T > 10000$K by $z = 800$, so Compton scattering off of hot electrons causes strong spectral distortions in the cosmic microwave background. The models in GO give a Compton y-parameter between 0.96×10^{-4} and 3.1×10^{-4}, and are thus all ruled out by the most recent observational constraint from the COBE FIRAS experiment, $y < 2.5 \times 10^{-5}$ (Mather *et al.* 1993).

There are essentially four mechanisms that can heat the IGM sufficiently to produce Compton y-distortions:

- Photoionization heating from UV photons (Shapiro & Giroux 1987, Donahue & Shull 1991)
- Compton heating from UV photons
- Mechanical heating from supernova-driven winds (Schwartz *et al.* 1975, Ikeuchi 1981, Ostriker & Cowie 1981)
- Cosmic ray heating (Ginzburg & Ozernoi 1965)

The second effect tends to drive the IGM temperature towards two-thirds of the temperature of the ionizing radiation, whereas the first effect tends to drive the temperature towards a lower value T^* that will be defined below. The third and fourth effect can produce much higher temperatures, often in the millions of degrees. The higher the temperature, the greater the y-distortion.

In the GO models, the second effect dominates, which is why they fail so badly. In this paper, we wish to place limits that are virtually impossible to evade. Thus we will use the most cautions assumptions possible, and assume that the latter three heating mechanisms are negligible.

2. THE COMPTON y-PARAMETER

Thomson scattering between CMB photons and hot electrons affects the spectrum of the CMB. It has long been known that hot ionized IGM causes spectral distortions to the CMB, known as the Sunyayev-Zel'dovich effect. A useful measure of this distortion is the Comptonization y-parameter (Kompanéets 1957, Sunyayev ans Zel'dovich 1969, Stebbins & Silk 1986, Bartlett & Stebbins 1991)

$$y = \int \left(\frac{kT_e - kT_\gamma}{m_e c^2}\right) n_e \sigma_t c\, dt = y^* \int \frac{(1+z)}{\sqrt{1+\Omega_0 z}} \Delta T_4(z) x(z) dz, \tag{1}$$

where

$$y^* \equiv \left[1 - \left(1 - \frac{x_{He}}{4x}\right) Y\right] \left(\frac{k \times 10^4\mathrm{K}}{m_e c^2}\right) \left(\frac{3 H_0 \Omega_{igm} \sigma_t c}{8\pi G m_p}\right) \approx 9.58 \times 10^8 h \Omega_{igm}.$$

Here T_e is the electron temperature, T_γ is the CMB temperature, $\Delta T_4 \equiv (T_e - T_\gamma)/10^4\mathrm{K}$, Ω_{igm} is the fraction of critical density in intergalactic medium, and $x(z)$ is the fraction of the hydrogen that is ionized at redshift z. Note that we may have $\Omega_{igm} \ll \Omega_b$, *i.e.* all baryons may not be in diffuse form. The integral is to be taken from the reionization epoch to today. In estimating the electron density n_e, we have taken the mass fraction of helium to be $Y \approx 24\%$ and assumed $x_{He} \approx x$, *i.e.* that helium never becomes doubly ionized and that the fraction that is singly ionized equals the fraction of hydrogen that is ionized. The latter is a very crude approximation, but makes a difference of only 6%.

Let us estimate this integral by making the approximation that the IGM is cold and neutral until a redshift z_{ion}, at which it suddenly becomes ionized, and after which it remains completely ionized with a constant temperature T_e. Then for $z_{ion} \gg 1$ and $T_e \gg z_{ion} \times 2.7\mathrm{K}$ we obtain

$$y \approx 6.4 \times 10^{-8} h \Omega_{igm} \Omega_0^{-1/2} T_4\, z_{ion}^{3/2},$$

where $T_4 \equiv T_e/10^4\mathrm{K}$. Substituting the most recent observational constraint from the COBE FIRAS experiment, $y < 2.5 \times 10^{-5}$ (Mather *et al.* 1993), into this expression yields

$$z_{ion} < 554 T_4^{-2/3} \Omega_0^{1/3} \left(\frac{h\Omega_{igm}}{0.03}\right)^{-2/3}. \tag{2}$$

Thus the only way to have z_{ion} as high as 1100 is to have temperatures considerably below 10^4K. In the following section, we will see to what extent this is possible.

3. IGM EVOLUTION IN THE STRONG UV FLUX LIMIT

In this section, we will calculate the thermal evolution of IGM for which

- the IGM remains almost completely ionized at all times,
- the Compton y-distortion is minimized given this constraint.

3.1. The ionization fraction

In a homogeneous IGM at temperature T exposed to a density of ζ UV photons of energy $h\nu > 13.6\,\mathrm{eV}$ per proton, the ionization fraction x evolves as follows:

$$\frac{dx}{d(-z)} = \frac{1+z}{\sqrt{1+\Omega_0 z}}\left[\lambda_{pi}(1-x) + \lambda_{ci}x(1-x) - \lambda_{rec}x^2\right], \tag{3}$$

where $H_0^{-1}(1+z)^{-3}$ times the rates per baryon for photoionization, collisional ionization and recombination are given by

$$\begin{cases} \lambda_{pi} \approx 1.04\times 10^{12}\,[h\Omega_{igm}\sigma_{18}]\,\zeta, \\ \lambda_{ci} \approx 2.03\times 10^4 h\Omega_{igm}T_4^{1/2}e^{-15.8/T_4}, \\ \lambda_{rec} \approx 0.717 h\Omega_{igm}T_4^{-1/2}\left[1.808 - 0.5\ln T_4 + 0.187 T_4^{1/3}\right], \end{cases} \tag{4}$$

and $T_4 \equiv T_e/10^4\mathrm{K}$. Here σ_{18} is the spectrally-averaged photoionization cross section in units of $10^{-18}\mathrm{cm}^2$. The differential cross section is given by (Osterbrock, 1974)

$$\frac{d\sigma_{18}}{d\nu}(\nu) \approx \begin{cases} 0 & \text{if } \nu < 13.6\,\mathrm{eV}, \\ 6.30\frac{e^{4-4\arctan(\epsilon)/\epsilon}}{\nu^4\left(1-e^{-2\pi/\epsilon}\right)} & \text{if } \nu \geq 13.6\,\mathrm{eV}, \end{cases} \tag{5}$$

where

$$\epsilon \equiv \sqrt{\frac{h\nu}{13.6\,\mathrm{eV}} - 1}.$$

The recombination rate is the total to all hydrogenic levels (Seaton 1959, Spitzer 1968). Recombinations directly to the ground state should be included here, since as will become evident below, the resulting UV photons are outnumbered by the UV photons that keep the IGM photoionized in the first place, and thus can be neglected when determining the equilibrium temperature.

At high redshifts, the ionization and recombination rates greatly exceed the expansion rate of the universe, and the ionization level quickly adjusts to a quasi-static equilibrium value for which the expression in square brackets in equation (3) vanishes. In the absence of photoionization, an ionization fraction x close to unity requires $T_e > 15000\mathrm{K}$. Substituting this into equation (2) gives consistency with $z_{ion} > 1000$ only if $h\Omega_{igm} < 0.008$, a value clearly inconsistent with the standard nucleosynthesis constraints (Smith *et al.* 1993). Thus any reheating scenario that relies on collisional ionization to keep the IGM ionized at all times may be considered ruled out by the COBE FIRAS data.

However, this does not rule out all ionized universe scenarios, since photoionization can achieve the same ionization history while causing a much smaller y-distortion. The lowest temperatures (and hence the smallest y-distortions) compatible with high ionization will be obtained when the ionizing flux is so strong that $\lambda_{pi} \gg \lambda_{ci}$. In this limit, to a good approximation, equation (3) can be replaced by the following simple model for the IGM:

- It is completely ionized ($x = 1$).
- When a neutral hydrogen atom is formed through recombination, it is instantly photoionized again.

Thus the only unknown parameter is the IGM temperature T_e, which determines the recombination rate, which in turn equals the photoionization rate and thus determines the rate of heating.

3.2. The spectral parameter T^*

The net effect of a recombination and subsequent photoionization is to remove the kinetic energy $\frac{3}{2}kT$ from the plasma and replace it with the kinetic energy $\frac{3}{2}kT^*$, where T^* is defined by $\frac{3}{2}kT^* \equiv \langle E_{uv}\rangle - 13.6\,\mathrm{eV}$ and $\langle E_{uv}\rangle$ is the average energy of the ionizing photons. Thus the higher the recombination rate, the faster this effect will tend to push the temperature towards T^*.

The average energy of the ionizing photons is given by the spectrum $P(\nu)$ as $\langle E_{uv}\rangle = h\langle\nu\rangle$, where

$$\langle\nu\rangle = \frac{\int_0^\infty P(\nu)\sigma(\nu)d\nu}{\int_0^\infty \nu^{-1}P(\nu)\sigma(\nu)d\nu}.$$

Here σ is given by equation (5). Note that, in contrast to certain nebula calculations where all photons get absorbed sooner or later, the spectrum should be weighted by the photoionization cross section. This is because most photons never get absorbed, and all that is relevant is the energy distribution of those photons that do. Also note that $P(\nu)$ is the energy distribution (W/Hz), not the number distribution which is proportional to $P(\nu)/\nu$.

Table 1. Spectral parameters

UV source	Spectrum $P(\nu)$	$\langle E_{uv}\rangle$	T^*
O3 star	T = 50000K Planck	17.3 eV	28300K
O6 star	T = 40000K Planck	16.6 eV	23400K
O9 star	T = 30000K Planck	15.9 eV	18000K
Pop. III star	T = 50000K Vacca	18.4 eV	36900K
Black hole, QSO	α = 1 power law	18.4 eV	37400K
	α = 2 power law	17.2 eV	27800K
	α = 0 power law	20.9 eV	56300K
	T = 100000K Planck	19.9 eV	49000K

The spectral parameters $\langle E_{uv}\rangle$ and T^* are given in Table 1 for some selected spectra. A power law spectrum $P(\nu) \propto \nu^{-\alpha}$ with $\alpha = 1$ fits observed QSO spectra rather well in the vicinity of the Lyman limit (Cheney & Rowan-Robinson 1981, O'Brien *et al.* 1988), and is also consistent with the standard model for black hole accretion. A Planck spectrum $P(\nu) \propto \nu^3/\left(e^{h\nu/kT} - 1\right)$ gives a decent prediction of T^* for stars with surface temperatures below 30000K. For very hot stars, more realistic spectra (Vacca 1993) fall off much slower above the Lyman limit, thus giving higher values of T^*. As seen in Table 1, an extremely metal poor star of surface temperature 50000K gives roughly the same T^* as QSO radiation. The only stars that are likely to be relevant to early photoionization scenarios are extremely hot and short-lived ones, since the universe is less than a million years old at $z = 1000$, and fainter stars would be unable to inject enough energy in so short a time. Conceivably, less massive stars could play a the dominant role later on, thus lowering T^*. However, since they radiate such a small fraction of their energy above the Lyman limit, very large numbers would be needed, which could be difficult to reconcile with the absence of observations of Population III stars today.

3.3. The thermal evolution

At the low temperatures involved, the two dominant cooling effects† are Compton drag against the microwave background photons and cooling due to the adiabatic expansion of the universe. Combining

† Another cooling mechanism is collisional excitation of atomic hydrogen followed by radiative de-excitation, which cools the IGM at a rate of (Dalgarno & McCray 1972)

$$\mathcal{H}_{ce} \approx 7.5 \times 10^{-19} e^{-11.8/T_4} n^2 (1-x)x\,\mathrm{erg\,cm^{-3}s^{-1}}.$$

The ratio of this cooling rate to the Compton cooling rate is

$$\frac{\mathcal{H}_{ce}}{\mathcal{H}_{comp}} \approx \frac{\exp\left[9.2 - 11.8/T_4\right](1-x)}{(1+z)T_4} h^2\Omega_{igm},$$

these effects, we obtain the following equation for the thermal evolution of the IGM:

$$\frac{dT}{d(-z)} = -\frac{2}{1+z}T + \frac{1+z}{\sqrt{1+\Omega_0 z}}\left[\lambda_{comp}(T_\gamma - T) + \frac{1}{2}\lambda_{rec}(T)(T^* - T)\right], \tag{6}$$

where

$$\lambda_{comp} = \frac{4\pi^2}{45}\left(\frac{kT_\gamma}{\hbar c}\right)^4 \frac{\hbar\sigma_t}{H_0 m_e}(1+z)^{-3} \approx 0.00417h^{-1}(1+z)$$

is $(1+z)^{-3}$ times the Compton cooling rate per Hubble time and $T_\gamma = T_{\gamma_0}(1+z)$. The factor of $\frac{1}{2}$ in front of λ_{rec} is due to the fact that the photoelectrons end up sharing their energy with the protons. We have taken $T_{\gamma_0} \approx 2.726$K (Mather *et al.* 1993). Numerical solutions to this equation are shown in Figure 1, and the resulting y-parameters are given in Table 2.

Table 2. Compton y-parameters for various scenarios

Model	Ω_0	Ω_{igm}	h	T^*	z_{ion}	$y/2.5\times10^{-5}$	FIRAS verdict
QSO BDM I	0.15	0.15	0.8	37400K	1100	6.30	Ruled out
QSO BDM II	0.15	0.15	0.8	37400K	200	1.43	Ruled out
O9 BDM	0.15	0.15	0.8	18000K	800	2.91	Ruled out
QSO BDM III	0.15	0.04	0.8	37400K	1100	0.67	OK
QSO CDM I	1	0.06	0.5	37400K	1100	0.17	OK
QSO CDM II	1	0.03	0.8	37400K	1100	0.16	OK

The temperature evolution separates into two distinct phases. In the first phase, which is almost instantaneous due to the high recombination rates at low temperatures, T rises very rapidly, up to a quasi-equilibrium temperature slightly above the temperature of the microwave background photons. After this, in the second phase, T changes only slowly, and is approximately given by setting the expression in square brackets in equation (6) equal to zero. This quasi-equilibrium temperature is typically much lower than T^*, since Compton cooling is so efficient at the high redshifts involved, and is given by

$$\Delta T \equiv T_e - T_\gamma \approx \frac{\lambda_{rec}}{2\lambda_{comp}}(T^* - T_e) \propto \frac{1}{1+z}g(T_e)h^2\Omega_{igm}(T^* - T), \tag{7}$$

independent of Ω_0, where $g(T_e) \propto\!\!\!\sim T^{-0.7}$ encompasses the temperature dependence of λ_{rec}. We typically have $T \ll T^*$. Using this, making the crude approximation of neglecting the temperature dependence of λ_{rec}, and substituting (7) into (1) indicates that

$$y \propto\!\!\!\sim h^3\Omega_{igm}^2\Omega_0^{-1/2}T_4^* z_{ion}^{1/2}.$$

Numerically selecting the best power-law fit, we find that this is indeed not too far off: the approximation

$$y \approx 0.0012h^{2.4}\Omega_{igm}^{1.8}\Omega_0^{-1/2}(T_4^*)^{0.8}(z_{ion}/1100)^{0.9} \tag{8}$$

is accurate to about 10% within the parameter range of cosmological interest. We have used equation (8) in Figure 3 by setting $y = 2.5\times10^{-5}$ and $z_{ion} = 1100$. The shaded region of parameter space is thus ruled out by the COBE FIRAS experiment for fully ionized scenarios.

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a quantity which is much smaller than unity for any reasonable parameter values when $T < 10^4$K. As will be seen, the temperatures at $z \approx 1000$ are typically a few thousand K, which with $h^2\Omega_{igm} < 0.1$ and $x > 0.9$ renders collisional excitation cooling more than nine orders of magnitude weaker than Compton cooling. Hence we can safely neglect collisional excitations when computing the IGM temperature, the reason essentially being that the temperatures are so low that this process is suppressed by a huge Boltzman factor.

4. CONCLUSIONS

A reanalysis of the Compton y-distortion arising from early reionization shows that despite the radical sharpening of the FIRAS limit on y, one still cannot conclusively assert that the universe underwent a neutral phase. Non-recombining scenarios where the ionization is thermal, caused by IGM temperatures exceeding 10^4K, are clearly ruled out. Rather, the loophole is for the dominant ionization mechanism to be photoionization. We have shown that for spectra characteristic of both QSO radiation and massive metal-poor stars, the resulting IGM temperatures are so low that typical CDM models with no recombination can still survive the FIRAS test by a factor of six. This conclusion is valid if the flux of ionizing radiation is not so extreme that Compton heating becomes important. This is not difficult to arrange, as the cross section for Thomson scattering is some six orders of magnitude smaller than that for photoionization.

For BDM models, the constraints are sharper. Non-recombining "classical" BDM models with $\Omega_{igm} = \Omega_0 \approx 0.15$ are ruled out even with the extremely cautious reheating assumptions used in this paper, the earliest ionization redshift allowed being $z \approx 130$. Such models involving early non-linear seeds that on energetic grounds can very plausibly provide a photoionization source capable of reionizing the universe soon after the period of first recombination inevitably generate Compton distortions of order 10^{-4}. These include texture as well as BDM models, both of which postulate, and indeed require, early reionization ($z > 100$) to avoid the generation of excessive anisotropy in the cosmic microwave background on degree angular scales.

Thus BDM models with reionization at $z \approx 1,000$ can survive the y test only if most of the baryons form BDM when reionization occurs, and are thereby removed as a source of y-distortion, at least in the diffuse phase. This may be difficult to arrange at $z > 100$, since once the matter is reionized at this high a redshift, Compton drag is extremely effective in inhibiting any further gas collapse until $z < 100$. Since it takes only a small fraction of the baryons in the universe to provide a source of photons sufficient to maintain a fully ionized IGM even at $z \sim 1000$, we suspect that most of the baryons remain diffuse until Compton drag eventually becomes ineffective. Moreover, the possibility that the IGM is only partially reionized at $z \sim 1000$ (*e.g.* GO), a situation which allows a lower value of the y-parameter, seems to us to be implausible as a delicate adjustment of ionization and recombination time-scales over a considerable range in z would be required. A complementary argument that greatly restricts the parameter space allowable for fully ionized BDM models appeals to temperature fluctuations induced on the secondary last scattering surface, both by first order Doppler terms on degree scales and by second order terms on subarcminute scales (Hu, Scott and Silk 1993). Thus, BDM models would seem to be in some difficulty because of the low limit on a possible y-distortion.

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