UNCORRELATED MEASUREMENTS OF THE CMB POWER SPECTRUM

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We describe how to compute estimates of the power spectrum C_ℓ from Cosmic Microwave Background (CMB) maps that not only retain all the cosmological information, but also have uncorrelated error bars and well-behaved window functions. We apply this technique to the 4-year COBE/DMR data.

Accurate future measurements of the angular power spectrum C_ℓ of the CMB would allow us to measure many key cosmological parameters with unprecedented accuracy [1]. It has recently been shown [2] how to compute a vector of power spectrum estimates $\mathbf{y}$ retaining all the cosmological information from a CMB map, and whose mean and covariance is given by

$$\begin{aligned} \langle \mathbf{y} \rangle &= \mathbf{F}\mathbf{c}, \\ \langle \mathbf{y}\mathbf{y}^t \rangle - \langle \mathbf{y} \rangle \langle \mathbf{y}^t \rangle &= \mathbf{F}. \end{aligned} \tag{1}$$

Here $\mathbf{c}$ is the vector of true power coefficients, i.e., $\mathbf{c}_\ell = C_\ell$, so the window function matrix and the covariance matrix are one and the same, equaling $\mathbf{F}$, the *Fisher information matrix* [2]. There are infinitely many ways of producing uncorrelated power estimates [3]. Making a factorization $\mathbf{F} = \mathbf{M}\mathbf{M}^t$ for some matrix $\mathbf{M}$, the new power estimates in the vector defined by

$$\hat{\mathbf{c}} \equiv \mathbf{M}^{-1}\mathbf{y} \tag{2}$$

will be uncorrelated, since $\langle \hat{\mathbf{c}}\hat{\mathbf{c}}^t \rangle - \langle \hat{\mathbf{c}} \rangle \langle \hat{\mathbf{c}}^t \rangle = \mathbf{I}$. $\langle \hat{\mathbf{c}} \rangle = \mathbf{M}^t\mathbf{c}$, so the new window function matrix will be $\mathbf{M}^t$. However, whereas the window functions of the original power estimates (the rows of $\mathbf{F}$) are always well-behaved (they are always non-negative [2], and are generally quite narrow, as shown in the top panel of Figure 1), there is no guarantee that the same will hold for the new window functions. As was recently shown [3], however, one generally obtains beautiful window functions if one requires $\mathbf{M}$ to be lower-triangular, in which case $\mathbf{F} = \mathbf{M}\mathbf{M}^t$ corresponds to a Cholesky decomposition. For the COBE/DMR case

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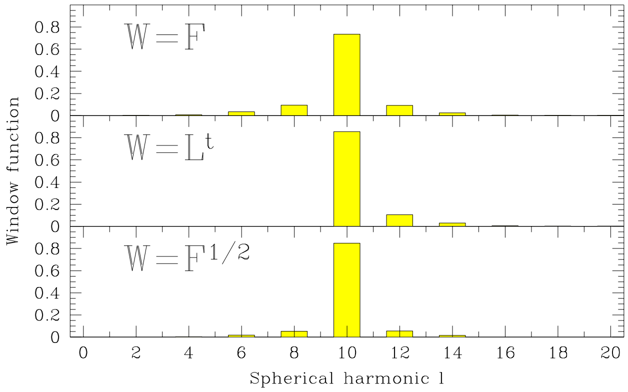

Figure 1: The $\ell_* = 10$ window functions for the three methods discussed, corresponding to using the original $\mathbf{y}$, Cholesky decomposing the Fisher matrix and taking its square root, respectively.

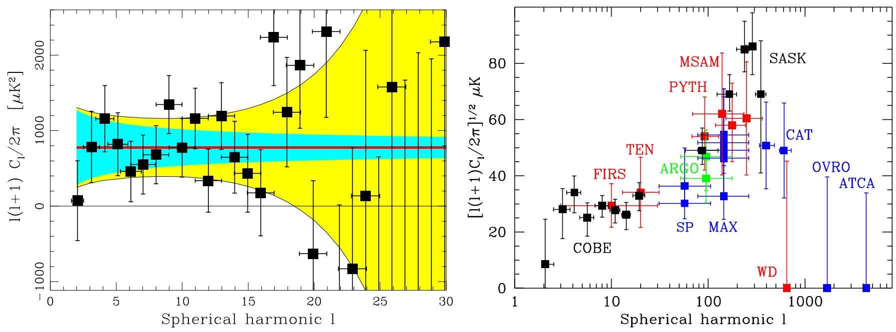

Figure 2: The power spectrum observed by COBE/DMR alone (left) and binned into 8 bands and compared with other experiments (right).

with a "custom" [4] Galaxy cut, this gives the narrow and non-negative window functions in the middle panel of Figure 1, with side lobes only to the right. Similarly, one could obtain window functions with side lobes only to the left by chosing $\mathbf{M}$ upper-triangular. A third choice, which is the one we recommend, is choosing $\mathbf{M}$ *symmetric*, which we write as $\mathbf{M} = \mathbf{F}^{1/2}$. The square root of the Fisher matrix is seen to give beautifully symmetric window functions (Figure 1, bottom) that are not only non-negative, but also even narrower than the original (top), which has roughly the bottom profile convolved with itself.

Figure 2 (left) shows the power spectrum extracted from the 4 year COBE data[4] with the minimum-variance method[2] and de-correlated with $\mathbf{M} = \mathbf{F}^{1/2}$. The error bars are for a flat 18 μK spectrum. These 29 data points thus contain all the cosmological information from COBE, distilled into 29 mutually exclusive (uncorrelated) and collectively exhaustive (jointly retaining all infor-

Table 1: The COBE power spectrum $\delta T \equiv [\ell(\ell+1)C_\ell/2\pi]^{1/2}$ in μK.

Band	ℓ_*	$\langle\ell\rangle$	$\Delta\ell$	δT	-1σ	$+1\sigma$
1	2	2.1	0.5	8.5	0	24.5
2	3	3.1	0.6	28.0	17.7	35.5
3	4	4.1	0.7	34.0	26.8	40.0
4	5-6	5.6	0.9	25.1	18.5	30.4
5	7-9	8.0	1.3	29.4	25.3	33.0
6	10-12	10.9	1.3	27.7	23.2	31.6
7	13-16	14.3	2.5	26.1	20.9	30.5
8	17-30	19.4	2.8	33.0	27.6	37.6

mation about cosmological parameters) chunks. To reduce scatter, these have been binned into 8 bands in Table 1 and Figure 2 (right). The data and references for the other experiments plotted can be found in recent compilations [5,6].

This method can readily be applied to other CMB experiments [7] as well as galaxy surveys [3,8]. In comparison, previous CMB power spectrum estimation methods [9,10,11,12] all had the drawback of giving correlated errors.

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