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OBSERVATIONAL EVIDENCE FOR STOCHASTIC BIASING

Max Tegmark[1,2] and Benjamin C. Bromley[3]

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ABSTRACT

We show that the galaxy density in the Las Campanas Redshift Survey (LCRS) cannot be perfectly correlated with the underlying mass distribution since various galaxy subpopulations are not perfectly correlated with each other, even taking shot noise into account. This rules out the hypothesis of simple linear biasing, and suggests that the recently proposed stochastic biasing framework is necessary for modeling actual data.

Subject headings: galaxies: statistics — large-scale structure of universe

1. INTRODUCTION

Measurements of clustering in upcoming galaxy redshift surveys hold the potential of measuring cosmological parameters with great accuracy (Tegmark 1997; Goldberg & Strauss 1997), especially when complemented by measurements of the Cosmic Microwave Background (Eisenstein *et al.* 1998; Hu *et al.* 1998ab). Since such measurements are only as accurate as our understanding of biasing, there has been a recent burst of work on the relation between the distribution of galaxies and the underlying mass.

Dekel and Lahav (1998; Dekel 1997 §5.5; Lahav 1996 §3.1) have proposed a robust framework termed "stochastic biasing", which drops the assumption that the galaxy density $\rho_g(\mathbf{r})$ is uniquely determined by the matter density ρ. Writing the corresponding density fluctuations as $g \equiv \rho_g/\langle\rho_g\rangle - 1$ and $\delta \equiv \rho/\langle\rho\rangle - 1$, g is modelled as a function of δ plus a random term. It has been known since the outset (*e.g.*, Dressler 1980) that any deterministic biasing relation $g = f(\delta)$ must be complicated, depending on galaxy type. Even allowing this, however, deterministic biasing still implies that the peaks and troughs of the two fields must coincide spatially, which need not be the case. Restricting attention to second moments, all the information about both such stochasticity and nonlinearity (of $f(\delta)$) can be contained in a single new function r (Pen 1998; Tegmark & Peebles 1998, hereafter TP98). Grouping the densities into a two-dimensional vector

$$\mathbf{x} \equiv \begin{pmatrix} \delta \\ g \end{pmatrix} \quad (1)$$

and assuming nothing except translational invariance, its Fourier transform $\widehat{\mathbf{x}}(\mathbf{k}) \equiv \int e^{-i\mathbf{k}\cdot\mathbf{r}}\mathbf{x}(\mathbf{r})d^3r$ obeys

$$\langle \widehat{\mathbf{x}}(\mathbf{k})\widehat{\mathbf{x}}(\mathbf{k}')^\dagger \rangle = (2\pi)^3\delta^D(\mathbf{k}-\mathbf{k}') \begin{pmatrix} P(\mathbf{k}) & P_\times(\mathbf{k}) \\ P_\times(\mathbf{k}) & P_g(\mathbf{k}) \end{pmatrix} \quad (2)$$

for some 2×2 power spectrum matrix that we will denote $\mathbf{P}(\mathbf{k})$. Here P is the conventional power spectrum of the mass distribution, P_g is the power spectrum of the galaxies, and $P_\times$ is the cross spectrum. It is convenient to rewrite this covariance matrix as

$$\mathbf{P}(\mathbf{k}) = P(\mathbf{k}) \begin{pmatrix} 1 & b(\mathbf{k})r(\mathbf{k}) \\ b(\mathbf{k})r(\mathbf{k}) & b(\mathbf{k})^2 \end{pmatrix} \quad (3)$$

where $b \equiv (P_g/P)^{1/2}$ is the bias factor (the ratio of luminous and total fluctuations) and the new function $r \equiv P_\times/(PP_g)^{1/2}$ is the dimensionless correlation coefficient between galaxies and matter. The special case $r = 1$ gives the simple deterministic biasing relation $g = b\delta$, however, both b and r may generally depend on scale.

Since the function $r(k)$ is a cosmologically important quantity, it has received much recent attention. Pen (1998) has shown how it can be measured using redshift space distortions and nonlinear effects, Scherrer & Weinberg (1998) have computed r for a number of theoretical models, and Blanton *et al.* 1998 (hereafter B98) have estimated b and r from hydrodynamic simulations. TP98 have computed the time-evolution of bias in the linear regime, while Taruya *et al.* (1998ab) have generalized this result to the perturbatively non-linear case (see also Mo & White 1996; Mo *et al.* 1997; Matarrese *et al.* 1997; Bagla 1998; Catelan 1998ab; Colin *et al.* 1998; Lemsen & Sheth 1998; Moscardini *et al.* 1998; Porciani 1998; Wechsler *et al.* 1998). Some of these numerical and theoretical predictions have been borne out in observational data which indicate very high bias values ($b \sim 4-6$) at $z \sim 3$, decreasing rapidly with time (Giavalisco *et al.* 1998).

In light of all this activity, it would be timely to observationally measure $r(k)$. This is the topic of the present paper. Unfortunately, measuring r directly requires knowledge of the true matter distribution δ. Although this information in principle may be obtained from *e.g.* POTENT reconstruction from peculiar velocity measurements (Dekel *et al.* 1990), the quasilinear redshift-space method of Pen (1998) or gravitational lensing (Van Waerbeke 1998), it will likely require better data than is presently available since systematic errors in the estimate of δ masquerade as $r < 1$. We therefore adopt an indirect approach, based on the following simple idea. If two different types of galaxies are both perfectly correlated with the matter, then they must also be perfectly correlated with each other. If we can demonstrate imperfect correlations between two galaxy subsamples then we will have shown that $r < 1$ for at least one of them. We show that $r < 1$ at high statistical significance, and place quantitative limits on r in §3.

2. RULING OUT SIMPLE BIASING

We base our analysis on galaxies in the Las Campanas Redshift Survey (LCRS; Schectman *et al.* 1996) grouped

[1] Institute for Advanced Study, Princeton, NJ 08540
[2] Hubble Fellow; max@ias.edu
[3] Dept. of Physics, Univ. of Utah, Salt Lake City, UT 84112

into 6 types or "clans" according to the spectral classification scheme of Bromley *et al.* (1998ab), ranging from very early-type galaxies (clan 1) to late-type objects (clan 6). To reduce shot noise we define our clan 4 as the combination of the original clans 4, 5 and 6. The clustering properties of these clans vary in a systematic way, revealing a progression in the relative bias factors b – see also Santiago & Strauss (1992). B98 predict that r depends strongly on the formation epoch of a galaxy population, so one might suspect that some of the LCRS clans have $r<1$.

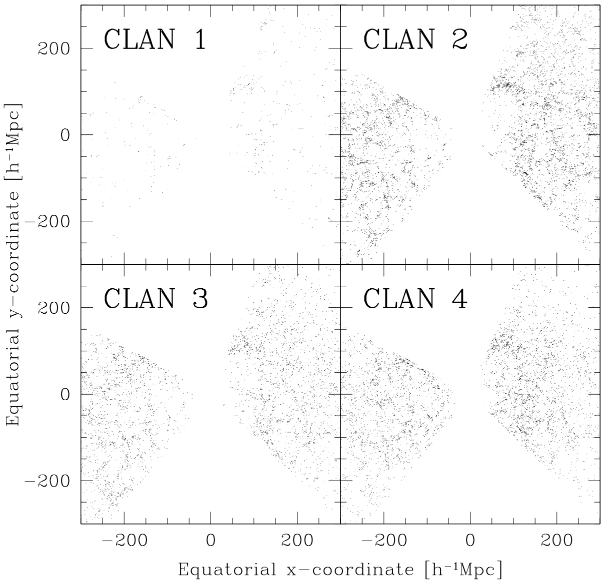

FIG. 1 — The four subsamples (clans) of the LCRS.

The LCRS consists of $n_f=327$ rectangular fields in the sky, and we further subdivide the volume into $n_s=10$ radial shells in the range 10 000 km/s$<cz<$45 000 km/s. Discarding galaxies outside of this redshift range (as in Lin *et al.* 1996) leaves 519, 8282, 5152 and 5669 galaxies in the four clans. For each of our $n\equiv n_f\times n_s$ spatial volumes V_α $(\alpha=1,...,n)$ and each clan $i=1,...,n_c$, we count the number of observed galaxies $N_\alpha^{(i)}$, and compute the expected number of galaxies $\bar N_\alpha^{(i)}$ using the selection function of Lin *et al.* (1996) with the clan-dependent Schechter parameters of Bromley *et al.* (1998a). We write the map of observed density fluctuations for the i^{th} clan as an n-dimensional vector $\mathbf{g}^{(i)}$, defined by $g_\alpha^{(i)}\equiv N_\alpha^{(i)}/\bar N_\alpha^{(i)}-1$. In a simple $r=1$ linear biasing model, we would have

$$g_\alpha^{(i)}=b_i\delta_\alpha+\varepsilon_\alpha^{(i)}, \tag{4}$$

where b_i is the bias of the i^{th} clan, δ_α is the matter density fluctuation in V_α and the shot noise contributions $\varepsilon_\alpha^{(i)}$ have zero mean and a diagonal covariance matrix

$$\mathbf{N}_{\alpha\beta}^{(i)}\equiv\langle\varepsilon_\alpha^{(i)}\varepsilon_\beta^{(i)}\rangle=\delta_{\alpha\beta}/\bar N_\alpha^{(i)}. \tag{5}$$

Can we rule this out? For a pair of clans i and j, consider the difference map

$$\Delta\mathbf{g}\equiv\mathbf{g}^{(i)}-f\mathbf{g}^{(j)}. \tag{6}$$

for different values of the factor f. If $f=b_i/b_j$, then equation (4) shows that the (unknown) matter density fluctuations δ_α will cancel out, and $\Delta\mathbf{g}$ will consist of mere shot noise whose covariance matrix is $\mathbf{N}\equiv\langle\Delta\mathbf{g}\Delta\mathbf{g}^t\rangle=\mathbf{N}^{(i)}+f^2\mathbf{N}^{(j)}$.

Given the alternative hypothesis that there is a residual signal with some covariance matrix $\mathbf{S}$, so that $\langle\Delta\mathbf{g}\Delta\mathbf{g}^t\rangle=\mathbf{N}+\mathbf{S}$, the most powerful "null-buster" test for ruling out the null hypothesis $\langle\Delta\mathbf{g}\Delta\mathbf{g}^t\rangle=\mathbf{N}$ is using the generalized χ^2-statistic (Tegmark 1998)

$$\nu\equiv\frac{\Delta\mathbf{g}^t\mathbf{N}^{-1}\mathbf{S}\mathbf{N}^{-1}\Delta\mathbf{g}-\mathrm{tr}\,\mathbf{N}^{-1}\mathbf{S}}{[2\mathrm{tr}\,\{\mathbf{N}^{-1}\mathbf{S}\mathbf{N}^{-1}\mathbf{S}\}]^{1/2}}, \tag{7}$$

which can be interpreted as the number of "sigmas" at which the noise-only null hypothesis is ruled out. We choose $\mathbf{S}_{\alpha\beta}=\xi(|\mathbf{r}_\alpha-\mathbf{r}_\beta|)$, where $\mathbf{r}_\alpha$ is the center of volume V_α and ξ is the correlation function measured by the LCRS (Tucker *et al.* 1997). We plot the results in Figure 2 for three pairs of clans i and j, and the corresponding valley-shaped curve tells us a number of things. The fact that $\nu\gg1$ on the left-hand-side (as $f\to0$, with all the weight on clan i) means that there is a strong detection of cosmological fluctuations above the shot noise level ($\nu\sim1$). Likewise, $\nu\gg1$ on the right-hand-side (as $f\to\infty$), which demonstrates cosmological signal in clan j. The fact that the curve dips for intermediate f-values tells us that the two density maps are correlated ($r>0$) and have common signal. The minimum is attained at the value f which gives the best fit relative bias b_i/b_j for this common signal. However, the fact that $\nu\gg1$ even at the minimum proves that even though some signal is shared in common, *not all of it is*: there are no values of b_i for which equation (4) can hold for any pair of clans i and j.

Note that Figure 2 does not directly tell us which clans are most correlated, since low minima can signal either strong correlations, low fluctuations or high shot noise — the latter being the case for the (rare) 1^{st} clan.

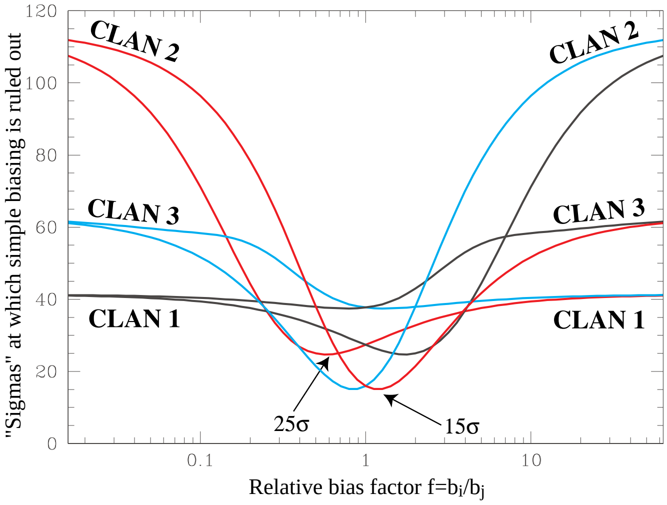

FIG. 2 — Significance at which we can rule out that different clan pairs are perfectly correlated (that $\mathrm{g}^{(i)}-f\mathrm{g}^{(j)}$ is pure shot noise).

3. MEASURING $r(k)$

Having ruled out non-stochastic linear biasing, *i.e.*, demonstrated that $r<1$ for some clans, let us now study in more detail what constraints can be placed on r.

3.1. *Upper limits on r*

Let us lengthen the 2-dimensional vector $\mathbf{x}=(\delta,g)$ from equation (1) to an $(1+n_c)$-dimensional vector including all n_c galaxy clans ($n_c=4$): $x_0=\delta$ and $x_i=g^{(i)}$, $i=1,...,n_c$, *i.e.*, $\mathbf{x}=(\delta,g^{(1)},g^{(2)},g^{(3)},g^{(4)})$. Next we factor the $(1+n_c)\times(1+n_c)$ covariance matrix $\mathbf{P}\equiv\langle\mathbf{x}\mathbf{x}^t\rangle$ as $\mathbf{P}_{ij}=\sigma_i\sigma_j\mathbf{R}_{ij}$, where $\sigma_i\equiv\mathbf{P}_{ii}^{1/2}$ is the rms fluctuation of the i^{th} component and $\mathbf{R}_{ij}\equiv\mathbf{P}_{ij}/\sigma_i\sigma_j$ is the dimensionless correlation matrix. The components $r_i\equiv R_{0i}$ clearly correspond to the correlations we discussed earlier, except that there is now one r for each clan, a vector of correlations $\mathbf{r}$. These are the numbers we are interested in, although all we can measure directly are the remaining degrees of freedom in $\mathbf{R}$, the galaxy-galaxy correlation matrix $\mathbf{Q}$ (the $n_c\times n_c$ submatrix in the lower right corner). What constraints can we place? Consider first the simpler case involving only $n_c=2$ two clans. Then

$$\mathbf{R}=\begin{pmatrix}1 & \mathbf{r}^t\\ \mathbf{r} & \mathbf{Q}\end{pmatrix}=\begin{pmatrix}1 & r_1 & r_2\\ r_1 & 1 & \rho\\ r_2 & \rho & 1\end{pmatrix}, \tag{8}$$

where ρ denotes the one number that we can measure: the correlation between clans 1 and 2. Since a correlation matrix cannot have negative eigenvalues, $\det\mathbf{R}\geq 0$. After some algebra, this gives the constraint

$$|r_1-\rho r_2|\leq\left[(1-\rho^2)(1-r_2^2)\right]^{1/2}. \tag{9}$$

Thus the larger we make r_2, the more tightly constrained r_1 becomes, and vice versa. If $r_2=1$, then the right hand side vanishes, forcing $r_1=\rho$. This gives an upper bound on the smaller of r_1 and r_2. The most weakly constrained case is clearly $r_1=r_2$, so we obtain

$$\min\{r_1,r_2\}\leq\left[\frac{1+\rho}{2}\right]^{1/2}. \tag{10}$$

For the case with $n_c=4$ clans, we get analogous inequalities for each of the $n_c(n_c+1)/2=10$ pairs of clans. Additional more complicated constraints follow from requiring a nonnegative determinant for all 4×4-submatrices and for the whole 5×5 matrix, although these will not be considered here.

Table 1. Correlations and relative bias for the four galaxy clans.

i	b_i/b_4	Galaxy correlation matrix $\mathbf{Q}$			
1	2.97±.04	1	.63±.02	.44±.02	.42±.03
2	1.28±.01	.63±.02	1	.88±.02	.81±.02
3	1.21±.01	.44±.02	.88±.02	1	.76±.03
4	1.00±.01	.42±.03	.81±.02	.76±.03	1

3.2. *Measuring the galaxy correlation matrix*

$\mathbf{Q}_{ij}=\mathbf{G}_{ij}/(\mathbf{G}_{ii}\mathbf{G}_{jj})^{1/2}$, where $\mathbf{G}$ is the $n_c\times n_c$ galaxy-galaxy covariance matrix (the bottom right submatrix of $\mathbf{P}$). Based on the analysis of Tegmark *et al.* (1998, hereafter T98), we compute an estimate $\mathbf{G}$ of the form $\mathbf{G}_{ij}\equiv{\mathbf{g}^{(i)}}^t\mathbf{E}\mathbf{g}^{(j)}$ minus shot noise, removed as in Appendix B of T98 by simply omitting self-pairs. The pair weighting is given by a matrix $\mathbf{E}\equiv\mathbf{N}^{-1}\mathbf{F}\mathbf{N}^{-1}$, where $\mathbf{N}$ is some typical shot noise covariance matrix – since it is different for each clan, we use the average $\mathbf{N}=n_c^{-1}\sum_i\mathbf{N}^{(i)}$ as a compromise. We make the choice $\mathbf{F}=\mathbf{I}$, which corresponds to measuring the *rms* fluctuations in the cells. Computing the window function W, defined by $\langle\mathbf{G}_{ij}\rangle=\int\mathbf{P}_{ij}(k)W(k)d^3k$, we find that $\mathbf{G}_{ij}$ mainly probes scales around $\lambda\equiv 2\pi/k\sim 10h^{-1}$Mpc.

Converting $\mathbf{G}$ back to $\mathbf{Q}$, we calculate error bars for its elements and all quantities derived below by generating 10^4 Monte Carlo realizations of Poisson shot noise for $N_\alpha^{(i)}$, adding this to the observed values and processing the result in exactly the same manner as the real data. Since our primary goal is to rule out $r_i=1$, we clearly do not care about sample variance in $\mathbf{Q}$: if biasing were simple, one would measure $r_i=1$ in all samples. When reading Tables 1 and 2, the reader should thus bear in mind that the error bars reflect shot noise only.

The results are shown in Table 1. The 2^{nd} column confirms that the clustering strength drops with clan number, as shown by Bromley *et al.* (1998a). All off-diagonal elements in the correlation matrix are seen to be significantly below unity, ranging from the 88% correlation between clans 2 and 3 to a mere 42% correlation between the earliest and latest galaxy types. Correlations generally decrease with separation in clan number, as expected. Applying equations (9) or (10) to this table gives a plethora of constraints on r_i. For example, equation (10) tells us that either r_1 or r_4 must be below $\sqrt{(1+0.42)/2}\approx 84\%$.

3.3. *The matter as a principal component*

Let $\lambda_1\geq\lambda_2\geq...\geq\lambda_{n_c}$ be the eigenvalues of $\mathbf{G}$, sorted from largest to smallest, with $\mathbf{e}_i$ the corresponding unit eigenvectors ($\mathbf{G}\mathbf{e}_i=\lambda_i\mathbf{e}_i$, $\mathbf{e}_i\cdot\mathbf{e}_j=\delta_{ij}$). It is instructive to decompose the fluctuation vector $\mathbf{x}$ into its uncorrelated principal components $y_i\equiv\mathbf{e}_i\cdot\mathbf{x}$:

$$\mathbf{x}=\sum_{i=1}^{n_c}\mathbf{e}_i y_i, \tag{11}$$

where $\langle y_iy_j\rangle=\delta_{ij}\lambda_i$. If biasing were simple and linear, we would have $\mathbf{x}=\mathbf{b}\,\delta$ for some vector $\mathbf{b}$ of bias values b_i. Equation (11) would therefore have only one term, with $\mathbf{e}_1\propto\mathbf{b}$ and $y_1\propto\delta$. Additional terms in equation (11) would correspond to random physical processes, uncorrelated with δ, that are pushing r_j below unity. If we want a model with a minimal amount of stochasticity, we should therefore interpret the first (largest) principal component as tracing the underlying matter distribution, *i.e.*, assume that $\delta=ay_1=a\mathbf{e}_1^t\mathbf{x}$ for some constant a. Since $\langle\mathbf{x}\mathbf{x}^t\rangle=\mathbf{G}$, this assumption gives $\langle\mathbf{x}\delta\rangle=a\langle\mathbf{x}\mathbf{x}^t\rangle\mathbf{e}_1=a\mathbf{G}\mathbf{e}_1=a\lambda_1\mathbf{e}_1$, $\langle\delta^2\rangle=a^2\mathbf{e}_1^t\langle\mathbf{x}\mathbf{x}^t\rangle\mathbf{e}_1=a^2\mathbf{e}_1^t\mathbf{G}\mathbf{e}_1=a^2\lambda_1$, and hence the correlation coefficients

$$r_j=\frac{\langle x_j\delta\rangle}{[\langle x_j\rangle^2\langle\delta\rangle^2]^{1/2}}=\sqrt{\frac{\lambda_1}{\mathbf{G}_{jj}}}(\mathbf{e}_1)_j. \tag{12}$$

These coefficients are shown in the last row of Table 2. The fraction of the variance of clan j that is caused by matter fluctuations is $\langle(\mathbf{e}_1)_j^2y_1^2\rangle/\langle x_j^2\rangle=(\mathbf{e}_1)_j^2\lambda_1/\mathbf{G}_{jj}=r_j^2$, simply the square of the correlation coefficient, so this is a useful way to interpret r_j.

Table 2 is indeed consistent with the hypothesis that the first principal component traces the matter: since all of its four components (boldface) have the same sign, we can interpret them as the (relative) bias factors that the clans would have if there were no randomness ($\lambda_2 = \lambda_3 = \lambda_4 = 0$). Since $(\mathbf{e}_1)_j \propto \mathbf{G}_{jj}^{1/2} \propto b_j r_j$, we can also interpret them as the best fit slopes in linear regressions of x_j against δ, the quantity that Dekel & Lahav (1998) call simply "b". The sharp decrease of $(\mathbf{e}_1)_j$ towards later clan types (their ratios are 5.1 : 1.7 : 1.3 : 1) is thus caused by the joint decrease of both the *measured* relative bias b_j (the 2^{nd} column of Table 1) and the correlation r_j .

Table 2. Principal components decomposition of the fluctuations.

i	λ_i	Components of eigenvector $\mathbf{e}_i$			
1	.564±.013	**.91±.01**	**.31±.01**	**.23±.01**	**.18±.01**
2	.122±.004	-.41±.01	.51±.01	.60±.01	.46±.02
3	.016±.002	.01±.02	-.08±.13	-.56±.16	.82±.15
4	.008±.001	-.08±.01	.80±.03	-.52±.10	-0.28±.14
Clan correlation r_j		.98±.002	.77±.02	.60±.02	.57±.03

4. CONCLUSIONS

Our basic conclusion is that bias is complicated. Rather than being describable by a single constant b_i for the i^{th} galaxy type, evidence is mounting that bias is

1. stochastic and/or nonlinear, requiring a 2^{nd} quantity r_i to characterize 2^{nd} moments,
2. scale dependent (b_i and r_i depend on k),
3. time-dependent (b_i and r_i depend on redshift).

Complication (1) was predicted by Dekel & Lahav (1998), and we have observationally confirmed it here. Complication (2) has long been observed on small scales (see *e.g.* Mann *et al.* 1996, B98 and references therein), while Complication (3) was predicted by Fry (1996) and TP98 and is gathering support from both simulations (Katz *et al.* 1998; B98) and observations (Giavalisco *et al.* 1998).

However, this is not cause for despair. The scale dependence is predicted to abate on large scales (Scherrer & Weinberg 1998), and the time-evolution due to gravity is calculable (TP98). As long as we limit ourselves to second moments (power spectra *etc.*), stochasticity merely augments $P_i(k)$ and $b_i(k)$ with an additional function $r_i(k)$. Furthermore, our constraints on r_i using galaxy data alone — without knowledge the underlying mass distribution — suggest strong regularities: The more similar two morphological types are, the stronger they are correlated.

More strikingly, our tentative identification of the 1^{st} principal component of the galaxy covariance matrix as the underlying matter distribution is in excellent agreement with the recent simulations of B98: galaxies with high bias are almost perfectly correlated with matter (we find $r_1 \sim 98\%$), whereas the less biased populations have weaker correlations (we find $r \sim 60\% - 80\%$). Thus matter clustering explains only $r^2 \sim 40\% - 60\%$ of the variance of these galaxies.

This analysis is merely a first step towards observationally measuring the parameters of stochastic biasing. It remains to explore the scale-dependence and time-evolution of r as well as probing higher-order moments of the joint distribution of galaxy types (e.g., Lahav & Saslaw 1992). However, our success in constraining r in the absence of difficult mass measurements is an encouraging indication for the analyses of upcoming galaxy surveys: With the techniques presented here, along with theoretical and numerical modeling, it may be possible to understand bias well enough to realize the full potential of these surveys for measuring fundamental cosmological parameters.

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