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# Bias and beyond

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**Abstract.** It is becoming increasingly clear that galaxy bias is complicated, with evidence indicating that it can be time-dependent, morphology-dependent, scale-dependent, non-linear and non-deterministic. We discuss strategies for overcoming these complications and performing precision cosmological tests with upcoming redshift surveys.

## 1 The Bad News

The relative distribution of galaxies and mass is of increasing concern in cosmology since constraints on cosmological parameters from galaxy surveys [15, 7, 9, 4] are only as accurate as our understanding of bias. Unfortunately, bias is complicated. The commonly used assumption that the matter density fluctuations $\delta(\mathbf{r})$ and the galaxy number density fluctuations $g(\mathbf{r})$ obey

$$g(\mathbf{r}) = b\,\delta(\mathbf{r}) \tag{1}$$

for some constant $b$ (the bias factor) appears to be violated in a number of ways. It has been known since the outset [3] that $b$ must depend on galaxy type. However, there is also evidence that it depends on scale (see *e.g.* [11, 1] and references therein) and on time. Time-evolution of bias is both expected theoretically [5, 16] and observed [6], with $b$ generally dropping towards unity from high values early on. This is due to two separate effects: (1) Early galaxies are more biased at birth since they tend to be associated with rarer peaks. (2) Old galaxies gradually debias as they are gravitationally pulled into the same overdensities as the dark matter (see Figure 1). Moreover, on small scales where non-linear dynamics is important, one expects nonlinear corrections to equation (1). Finally, there are good reasons to believe that there is *no* deterministic relation that can replace equation (1), but that bias is inherently somewhat stochastic [2, 12, 16, 1].

## 2 The Good News

Does this mean that galaxy redshift surveys will contribute nothing to the quest for accurate measurements of cosmological parameters? Not necessarily. Below we will describe an approach that may be able to take us beyond the murky marshes of bias and let us do precision cosmology without assuming any bias relation in place of equation (1).

Dekel and Lahav have proposed a framework termed *stochastic biasing* [2], which drops the assumption that the galaxy fluctuations $g(\mathbf{r})$ is uniquely determined by the matter fluctuations $\delta(\mathbf{r})$. Instead, $g$ is modeled as a function of $\delta$ plus a random term. Restricting attention to second moments, all the information about stochasticity is contained in a single new function $r(k)$ [12, 16]. Grouping the fluctuations into a two-dimensional vector

$$\mathbf{x} \equiv \begin{pmatrix} \delta \\ g \end{pmatrix} \tag{2}$$

and assuming nothing except translational invariance, its Fourier transform $\widehat{\mathbf{x}}(\mathbf{k}) \equiv \int e^{-i\mathbf{k}\cdot\mathbf{r}}\mathbf{x}(\mathbf{r})d^3r$ obeys

$$\langle \widehat{\mathbf{x}}(\mathbf{k})\widehat{\mathbf{x}}(\mathbf{k}')^\dagger \rangle = (2\pi)^3\delta^D(\mathbf{k}-\mathbf{k}') \begin{pmatrix} P(\mathbf{k}) & P_\times(\mathbf{k}) \\ P_\times(\mathbf{k}) & P_g(\mathbf{k}) \end{pmatrix} \tag{3}$$

for some $2\times 2$ power spectrum matrix that we will denote $\mathbf{P}(\mathbf{k})$. Here $P$ is the conventional power spectrum of the mass distribution, $P_g$ is the power spectrum of the galaxies, and $P_\times$ is the cross spectrum. It is convenient to rewrite this covariance matrix as

$$\mathbf{P}(\mathbf{k}) = P(\mathbf{k}) \begin{pmatrix} 1 & b(\mathbf{k})r(\mathbf{k}) \\ b(\mathbf{k})r(\mathbf{k}) & b(\mathbf{k})^2 \end{pmatrix} \tag{4}$$

where $b \equiv (P_g/P)^{1/2}$ is the bias factor (the ratio of galaxy and total fluctuations) and the new function $r \equiv P_\times/(PP_g)^{1/2}$ is the dimensionless correlation coefficient between galaxies and matter. Note that both $b$ and $r$ generally depend on scale $k$. The Schwarz inequality shows that the special case $r=1$ implies the simple deterministic equation (1), and the converse is of course true as well.

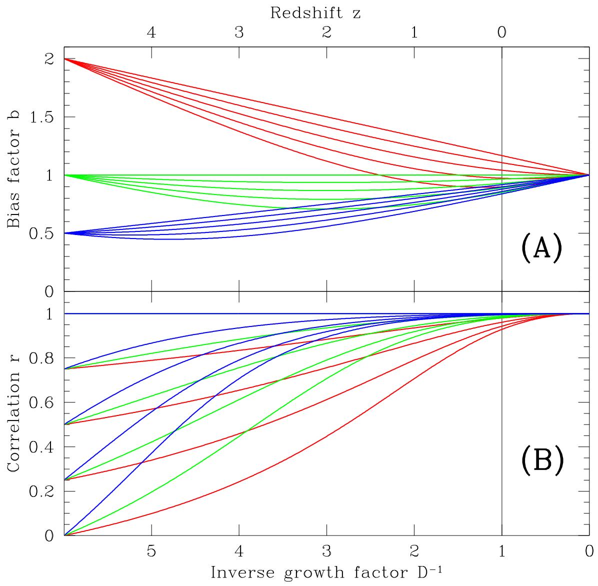
Since the function $r(k)$ is a cosmologically important quantity, it has received much recent attention. Pen has shown how it can be measured using redshift space distortions and nonlinear effects [12], and it has recently been computed for a number of theoretical models [13]. Its time-evolution has been derived in the linear regime [16] and generalized to the perturbatively non-linear case [14]. For a fixed galaxy population in an in an Einstein-de Sitter universe ($\Omega_m = 1$ and $\Omega_\Lambda = 0$, as in the standard cold dark matter model), the relation between the values $(b, r)$ at redshift $z$ and the present values $(b_0, r_0)$ reduces to [16]

$$b = [z^2 - 2z(1+z)b_0 r_0 + (1+z)^2 b_0^2]^{1/2}, \quad (5)$$

$$r = [(1+z)b_0 r_0 - z]/b, \quad (6)$$

plotted in Figure 1. Both $b$ and $r$ have now been measured using hydrodynamic simulations [1], and other groups are performing related studies.

**Figure 1.** The evolution of bias (A) and correlation (B) is shown for 15 models that at Einstein-de Sitter redshift $z = 5$ have bias $b = 0.5$, 1, and 2 and correlation coefficients $r = 0$, 0.25, 0.5, 0.75, and 1. In the top panel, $r$ increases upward in each quintuplet of lines. In the bottom panel, $b_0$ increases downward in each triplet of lines.




Why is this useful? How can one learn anything about the matter density without making assumptions about how it relates to the galaxy density? By assuming that the *velocities* are the same for galaxies as for other matter on large scales, *i.e.*, that they are both caused by gravity. This means that the characteristic clustering anisotropies known as redshift-space distortions [10, 8] will appear even if galaxies are completely uncorrelated. Within the stochastic biasing framework, Kaiser's classic result (which assumes $r = 1$) gets generalized to [12, 2]

$$P_{obs}(\mathbf{k}) = \left[b(k)^2 + 2\Omega_m^{0.6} b(k) r(k) x + \Omega_m^{1.2} x^2\right] P(\mathbf{k}), \quad (7)$$

where $x \equiv (\widehat{\mathbf{r}}\cdot\widehat{\mathbf{k}})^2$ and $\widehat{\mathbf{r}}$ is the line of sight direction (for the special case of a volume-limited distant survey). The right-hand side is a quadratic polynomial in $x$, so with a large data set such as the 2-degree Field Survey (2dF) or the Sloan Digital Sky Survey (SDSS), one can accurately measure the three coefficients. For each scale $k$, equation (7) thus gives us three independent equations. If the matter density $\Omega_m$ can be accurately measured by other means (using say upcoming CMB experiments, supernova 1a surveys and cluster abundance evolution), then these three equations are readily solved for the three unknowns $P(k)$, $b(k)$ and $r(k)$. The last two would then provide valuable insights about galalaxy formation, whereas $P(k)$ would give us the long sought for power spectrum of the underlying matter distribution which could, among other things, provide precision constraints on neutrino masses [9] and the Hubble constant [4].

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