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REIONIZATION IN AN OPEN CDM UNIVERSE: IMPLICATIONS FOR COSMIC MICROWAVE BACKGROUND FLUCTUATIONS[1]

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Abstract

We generalize previous work on early photoionization to CDM models with $\Omega < 1$. Such models have received recent interest because the excess power in the large–scale galaxy distribution is phenomenologically fit if the "shape parameter" $\Gamma \equiv h\Omega_0 \approx 0.25$. It has been argued that such models may require early reionization to suppress small-scale anisotropies in order to be consistent with experimental data. We find that if the cosmological constant $\lambda = 0$, the extent of this suppression is quite insensitive to Ω_0. Given a σ_8-normalization today, the loss of small–scale power associated with a lower Ω_0 is partially canceled by higher optical depth from longer lookback times and by structures forming at higher redshifts before the universe becomes curvature–dominated. The maximum angular scale on which fluctuations are suppressed decreases when Ω_0 is lowered, but this effect is also rather weak and unlikely to be measurable in the near future. For flat models, on the other hand, where $\lambda_0 = 1 - \Omega_0$, the negative effects of lowering Ω_0 dominate, and early reionization is not likely to play a significant role if $\Omega_0 \ll 1$. The same applies to CDM models where the effective Γ is lowered by increasing the number of relativistic particle species.

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1 Introduction

Early reionization of the universe would affect fluctuations in the cosmic microwave background radiation (CBR) on degree-scales and sub–degree scales. Considerable effort is now being devoted to measuring such fluctuations, so a detailed knowledge of the ionization history is desirable in order to give the experimental results their proper cosmological interpretation.

It is generally believed that the intergalactic medium (IGM) was reionized at some time in the past. The main reason for this is the absence of a Gunn-Peterson trough in the spectra of high redshift quasars (Gunn & Peterson 1965; Steidel & Sargent 1987; Webb *et al.* 1992), indicating that reionization occurred at least as early as $z = 4$. For CBR applications, the relevant question is not whether reionization occurred, but when it occurred. For instance, to provide an optical depth to scattering exceeding 20%, which suffices to reconcile CDM with all observational CBR limits (Sugiyama, Silk & Vittorio 1993), reionization must have occurred by redshift $z = 30$, adopting the nucleosynthesis value of the baryon density.

In Tegmark, Silk & Blanchard 1994 ("Paper 1"), early photoionization of the intergalactic medium was discussed in a fairly model–independent way, in order to investigate whether early structures corresponding to rare Gaussian peaks in a CDM model could photoionize the intergalactic medium sufficiently early to appreciably smooth out the microwave background fluctuations. In this paper, the results of Paper 1 will be generalized to $\Omega < 1$ models with non–zero cosmological constant Λ. Essentially all the notation used in this paper was defined in Paper 1 and, in the interest of brevity, some of the definitions will not be repeated here.

Just as in Paper 1, our basic picture is the following: An ever larger fraction f_s of the baryons in the universe falls into nonlinear structures and forms galaxies. A certain fraction of these baryons form stars or quasars which emit ultraviolet radiation. Some of this radiation escapes into the ambient intergalactic medium (IGM), which is consequently photoionized and heated. Due to cooling losses and recombinations, the net number of ionizations per UV photon, f_{ion}, is generally less than unity.

The results that we present here generalize the previous work to the case of an open universe. Lowering Ω_0 has four distinct effects:

1. Density fluctuations gradually stop growing once $z \lesssim \Omega_0^{-1}$. Thus given the observed power spectrum today, a lower Ω_0 implies that the first structures formed earlier.
2. Matter-radiation equality occurs later, which shifts the turning-point of the CDM power spectrum toward larger scales. This means less power on very small scales (such as $\sim 10^6 M_\odot$) relative to the scales at which we normalize the power spectrum (namely galaxy cluster scales, $\sim 8h^{-1}$Mpc or even the much larger COBE scale). One consequence is that the first structures form later.
3. The lookback time to a given ionization redshift becomes larger, resulting in a higher optical depth.
4. The horizon at a given ionization redshift subtends a smaller angle on the sky, thus lowering the angular scale below which CBR fluctuations are suppressed.

Thus in terms of the virialization redshift z_{vir} defined in Paper 1, the redshift at which typical structures to go nonlinear, effect 1 increases z_{vir} whereas effect 2 decreases z_{vir}. These two effects influence f_s, the fraction of baryons in nonlinear structures, in opposite directions. As to effect 2, it should be noted that this applies not only to CDM, but to any spectrum that "turns over" somewhere between the very smallest nonlinear scales ($\sim 10^6 M_\odot$) and the very largest ($\sim 10^{21} M_\odot$) scales at which we COBE-normalize. Yet another effect of lowering Ω_0 is that the ionization efficiency f_{ion} drops slightly, at most by a factor $\Omega_0^{1/2}$. This is completely negligible compared to the above-mentioned effects, as z_{ion} depends only logarithmically on the efficiency.

In the following sections, we will discuss each of these four effects in greater detail, and then compute their combined modification of the ionization history in a few scenarios.

2 The Boost Factor

When curvature and vacuum density are negligible, sub-horizon-sized density fluctuations simply grow as the scale factor $a \propto (1+z)^{-1}$. Thus at early times $z \gg \Omega_0^{-1}$, we can write

$$\delta = \frac{B(\Omega_0, \lambda_0)}{1+z}\delta_0$$

for some function B independent of z that we will refer to as the *boost factor*. Clearly $B(1,0) = 1$. Thus if certain structures are assumed to form when δ equals some fixed value, then given the observed power spectrum today, the boost factor tells us how much earlier these structures would form than they would in a standard flat universe. The boost factor is simply the inverse of the so called growth factor, and can be computed analytically for a number of special cases (see *e.g.* Peebles 1980). For the most general case, the fit

$$B(\Omega_0, \lambda_0) \approx \frac{2}{5\Omega_0}\left[\Omega_0^{4/7} - \lambda_0 + \left(1 + \frac{\Omega_0}{2}\right)\left(1 + \frac{\lambda_0}{70}\right)\right]$$

is accurate to within a few percent for all parameter values of cosmological interest (Carroll *et al.* 1992). The exact results are plotted in Figure 1 for the case $\lambda_0 = 0$ and the flat case $\lambda_0 = 1 - \Omega_0$. Since we will limit ourselves to these two cases, the simple power-law fits

$$\begin{cases} B(\Omega_0, 0) & \approx \Omega_0^{-0.63}, \\ B(\Omega_0, 1-\Omega_0) & \approx \Omega_0^{-0.21}, \end{cases}$$

which are accurate to within 1% for $0.2 \leq \Omega_0 \leq 1$, will suffice for our purposes. Note that the standard rule of thumb that perturbations stop growing at $1+z \approx \Omega_0^{-1}$, indicating $B \propto \Omega_0^{-1}$, is not particularly accurate in this context.

3 The Power Spectrum Shift

As mentioned above, lowering $h\Omega_0$ causes the first structures go nonlinear at a later redshift. This is quantified in the present section.

The standard CDM model with power-law initial fluctuations proportional to k^n predicts a power spectrum that is well fitted by (Bond & Efstathiou 1984; Efstathiou *et al.* 1992)

$$P(k) \propto \frac{q^n}{\left(1+\left[aq+(bq)^{1.5}+(cq)^2\right]^{1.13}\right)^{2/1.13}},$$

where $a \equiv 6.4$, $b \equiv 3.0$, $c \equiv 1.7$, $q \equiv (1h^{-1}\mathrm{Mpc})k/\Gamma$ and the "shape parameter" Γ will be discussed below. Although this fit breaks down for scales comparable to the curvature scale $r_{curv} = H_0^{-1}|1-\Omega_0|^{-1/2}$, it is quite accurate for the much smaller scales that will be considered in the present paper. Rather, its main limitation is that it breaks down if Ω_0 is so low that the baryon density becomes comparable to the density of cold dark matter. Thus for $\Omega_b \approx 0.05$, the results cannot be taken too seriously for $\Omega_0 < 0.2$. We will limit ourselves to the standard $n=1$ model here, as the tilted $(n<1)$ case was treated in Paper 1 and was seen to be be essentially unable to reionize the universe early enough to be relevant to CBR anisotropies. The same applies to models with mixed hot and cold dark matter.

Let us define the *amplitude ratio*

$$R(\Gamma, r_1, r_2) \equiv \frac{\sigma(r_1)}{\sigma(r_2)},$$

where $\sigma(r_1)$ and $\sigma(r_2)$ are the r.m.s. mass fluctuation amplitudes in spheres of radii r_1 and r_2, *i.e.*

$$\sigma(r)^2 \propto \int_0^\infty P(k)\left[\frac{\sin kr}{(kr)^3} - \frac{\cos kr}{(kr)^2}\right]^2 dk.$$

As in Paper 1, we normalize the power spectrum so that $\sigma(8h^{-1}\mathrm{Mpc})$ equals some constant denoted by σ_8, and M_c will denote the characteristic mass of the first galaxies to form. The corresponding comoving length scale r_c is given by $M_c = \frac{4}{3}\pi r_c^3\rho$, where ρ is the mean density of the universe. Thus given σ_8, what is relevant for determining when the first galaxies form is the amplitude ratio

$$R(\Gamma, r_c, 8h^{-1}\mathrm{Mpc}).$$

This ratio is computed numerically, and the results are plotted as a function of Γ in Figure 2 for a few different values of the cutoff mass M_c. It is easy to see why the amplitude ratio increases with Γ, since on a logarithmic scale, a decrease in Γ simply shifts the entire power spectrum towards lower k, thus decreasing the amount of power on very small scales relative to that on large scales. The fit

$$R(\Gamma, r_c, 8h^{-1}\mathrm{Mpc}) \approx 3 + 7.1\ln(1h^{-1}\mathrm{Mpc}/r_c)\Gamma$$

is accurate to within 10% for $0.05 < \Gamma < 2$ and $100\mathrm{pc} < r_c < 100\mathrm{kpc}$.

4 The Optical Depth

Since a lower Ω_0 implies a larger $|dt/dz|$ and an older universe, the optical depth out to a given ionization redshift z_{ion} is greater for small Ω_0. For a given ionization history $\chi(z)$, the optical depth for Thomson scattering is given by

$$\begin{cases} \tau(z) & = \tau^* \int_0^z \frac{(1+z')^2}{\sqrt{\lambda_0 + (1+z')^2(1-\lambda_0+\Omega_0 z')}}\chi(z')dz', \\ \tau^* & = \frac{3\Omega_b}{8\pi}\left[1 - \left(1 - \frac{\chi_{He}}{4\chi}\right) f_{He}\right] \frac{H_0 c\sigma_t}{m_p G} \approx 0.057h\Omega_b, \end{cases}$$

where we have taken the mass fraction of helium to be $f_{He} \approx 24\%$ and assumed $\chi_{He} \approx \chi$, *i.e.* that helium never becomes doubly ionized and that the fraction that is singly ionized equals the fraction of hydrogen that is ionized. The latter is a very crude approximation, but has the advantage that the error can never exceed 6%. If the universe is fully ionized for all redshifts below z, the integral can be done analytically for $\lambda_0 = 0$:

$$\tau(z) = \frac{2\tau^*}{3\Omega_0^2}\left[2 - 3\Omega_0 + (\Omega_0 z + 3\Omega_0 - 2)\sqrt{1+\Omega_0 z}\right] \approx 0.038\frac{h\Omega_b z^{3/2}}{\Omega_0^{1/2}} \quad (1)$$

for $z \gg \Omega_0^{-1}$. As is evident from the asymptotic behavior of the integrand, τ is independent of λ_0 in the high redshift limit. Thus optical depth of unity is attained if reionization occurs at

$$z \approx 92\left(\frac{h\Omega_b}{0.03}\right)^{-2/3}\Omega_0^{1/3}.$$

Besides fluctuation suppression, polarization of the CBR is another interesting probe of reionization (Crittenden, Davis & Steinhardt 1994). Also for this application, the optical depth is a key parameter.

5 The Angular Scale

It is well known that reionization suppresses CBR fluctuations only on angular scales below the horizon scale at last scattering. Combining the standard expressions for horizon radius (*e.g.* Kolb & Turner 1990) and angular size, this angle is given by

$$\theta = 2\tan^{-1}\left[\frac{sqrt1 + z\Omega_0^{3/2}/2}{\Omega_0 z - (2-\Omega_0)(\sqrt{1+\Omega_0 z} - 1)}\right]. \quad (2)$$

For $z \gg \Omega_0^{-1}$, this reduces to

$$\theta \approx \sqrt{\frac{\Omega_0}{z}}, \quad (3)$$

but as is evident from Figure 3, this is quite a bad approximation except for $z \gg 100$. If we substitute it into equation (1) nonetheless, to get a rough estimate, we conclude that optical depth unity is obtained at an epoch whose horizon scale subtends the angle

$$\theta \approx 12^\circ \left(\frac{h\Omega_b\Omega_0}{0.03}\right)^{1/3}, \tag{4}$$

i.e., the dependence on all three of these cosmological parameters is relatively weak. As discussed in Paper 1, fluctuations on angular scales much smaller than this are suppressed by a factor

$$P(z) \equiv 1 - e^{-\tau(z)},$$

the *opacity*, which is the probability that a photon was Thompson scattered after redshift z. Its derivative, the *visibility function* $f_z = dP/dz$, is the probability distribution for the redshift at which last scattering occurred, the profile of the last scattering surface. The *angular visibility function*

$$f_\theta(\theta) = \left|\frac{dP_s}{d\theta}\right| = \left|\frac{d\theta}{dz}\right|^{-1} \frac{dP_s}{dz}$$

is plotted in Figure 4 for the case where the universe never recombines (the curves for the more general case with reionization at some redshift z_{ion} can be read off from Figure 4 as described in Paper 1). These functions give a good idea of the range of angular scales on which suppression starts to become important. In plotting these curves, the exact expression (2) has been used, rather than the approximation (3). It is seen that the qualitative behavior indicated by equation (4) is correct: as Ω_0 is lowered, the peak shifts down toward smaller angular scales, but the Ω_0-dependence is quite weak.

6 Cosmological Consequences

We will now compare the effect of lowering Γ in three cosmological models. The first model, which will be referred to as "open CDM" for short, has

$\lambda_0 = 0$. The second model, referred to as "ΛCDM", has $\lambda_0 = 1 - \Omega_0$. The *shape parameter* essentially tells us how early the epoch of matter-radiation equality occurred, and is given by

$$\Gamma = h\Omega_0 \left(\frac{g_*}{3.36}\right)^{-1/2},$$

where $g_* = 3.36$ corresponds to the standard model with no other relativistic degrees of freedom than photons and three massless neutrino species. In open CDM and ΛCDM, we have $g_* = 3.36$, so that $\Omega_0 = \Gamma/h$. The third model, referred to as τCDM (Dodelson, Gyuk & Turner 1994), has $\lambda_0 = 0$ and $\Omega = 1$, and achieves a lower value of Γ by increasing g_* instead.

Including the effect of the boost factor, equation (9) in Paper 1 becomes

$$1 + z_{ion} = \frac{\sqrt{2}\sigma_8}{\delta_c}\, R(h\Omega_0, r_c, 8h^{-1}\mathrm{Mpc})\, B(\Omega_0, \lambda_0)\, \mathrm{erfc}^{-1}\left[\frac{1}{2f_{uvpp}f_{ion}}\right].$$

The ionization redshift z_{ion} is plotted as a function of the shape parameter in Figure 5 for the various scenarios specified in Table 1. It is seen that for the open model, the dependence on $\Omega_0 = \Gamma/h$ is typically much weaker than the dependence on other parameters. One reason for this is that changes in the boost factor and the amplitude ratio partially cancel each other. For ΛCDM, the Ω_0-dependence is stronger, since the boost factor is weaker. In the τCDM model, the dependence on Γ/h is even stronger, as there is no boost factor whatsoever to offset the change in the amplitude ratio.

The scenarios in Table 1 are similar to those in Paper 1. In the one labeled "very optimistic", the high value for f_{uvpp}, the net number of produced UV photons per proton, is obtained by assuming that the main source of ionizing radiation is black hole accretion rather than conventional stars. Note that this speculative assumption still only increases z_{ion} by $3.55/3.00 - 1 \approx 18\%$, the efficiency dependence being merely logarithmic.

Figure 6, in a sense the most important plot in this paper, shows the opacity as a function of Γ/h for the various scenarios. Because of the increase

	Pess.	Mid.	Opt.	Very opt.
σ_8	0.5	1	1.1	1.2
δ_c	2.00	1.69	1.44	1.33
h	0.5	0.5	0.8	0.8
$M_c[M_\odot]$	10^8	10^6	10^5	10^5
$f = f_{ion} f_{uvpp}$	1	120	23,000	10^6
$\mathrm{erfc}^{-1}[1/2f]$	0.48	2.03	3.00	3.55
$h^2\Omega_b$	0.010	0.013	0.015	0.020

Table 1: Parameters used

in optical depth due to larger lookback times, the open model now gives slightly larger opacities for lower $\Omega_0 = \Gamma/h$. However, this dependence is seen to be quite week. For ΛCDM, where the boost factor contributes less, the net result is seen to be the opposite; a slight decrease in the opacity for lower $\Omega_0 = \Gamma/h$. For the τCDM model, where there is neither a boost factor nor an increase in the lookback time, this drop in opacity is seen to be much sharper. Note that the dependence on other uncertain parameters, summarized by the four scenarios in Table 1, is quite strong. Indeed, this dependence is stronger than the effect of moderate changes in Γ/h, so in the near future, it appears unlikely that opacity limits will be able to constrain the shape parameter except perhaps in the τCDM model.

The τCDM situation is summarized in Figure 7. To attain at least 50% opacity, $h\Omega_b$ must lie above the heavy curve corresponding to the scenario in question. On the other hand, nucleosynthesis (Smith *et al.* 1993; Walker *et al.* 1991) places a strict upper bound on this quantity if we assume that $h \geq 0.5$[2] It is seen that a shape parameter as low as $\Gamma \simeq 0.25$, which would match large-scale structure observations (Peacock & Dodds 1994), is quite difficult to reconcile with these two constraints.

[2]The τCDM model also alters the nucleosynthesis process (Dodelson, Gyuk & Turner 1993), but this can only marginally relax the bounds unless the τ neutrino is in a mass range incompatible with τCDM (Gyuk & Turner 1994).

7 Discussion

Lowering Γ is an attractive resolution of the problem that arises in reconciling the observed structure in the universe on large scales with observations on megaparsec scales. The empirical power spectrum is well fit by $\Gamma \approx 0.25$ (Peacock & Dodds 1994). Kamionkowski & Spergel (1993) have found that primordial adiabatic fluctuations in an open universe with $\Omega \approx 0.3$ are reconcilable with large-scale CMB anisotropy. On degree scales Kamionkowski, Spergel & Sugiyama (1994) require reionization with optical depth $\tau \sim 1$ in order to reconcile the low density open model with recent experimental limits, if the lowest limits are adopted. In a low Ω_0 ΛCDM model, the situation is not so critical, but reionization is required if the lowest limits (SP91) are adopted on degree scales (Gaier *et al.* 1992); $\tau \sim 0.5$ suffices however. A similar but slightly more favorable situation occurs in a τCDM model, where $\Gamma = h\Omega_0(g_*/3.36)^{-1/2}$ is reduced by increasing g_* by a factor of ~ 4, but some reionization is still required to match SP91.

We have found that reionization giving τ in the range 0.5 to 1 is readily produced and even natural in open models. This is because of the early formation of structure in combination with the increased age of the universe, effects which compensate for the flattening of the power spectrum due to the delay in matter domination. However, the ΛCDM and τCDM models with low Ω_0 fare less well in this regard, since the loss of small-scale power is not balanced by significantly earlier structure formation. With Ω_b in the range given by standard nucleosynthesis, a significant optical depth $\tau \gtrsim 0.5$ is difficult to attain in either the ΛCDM or τCDM scenarios.

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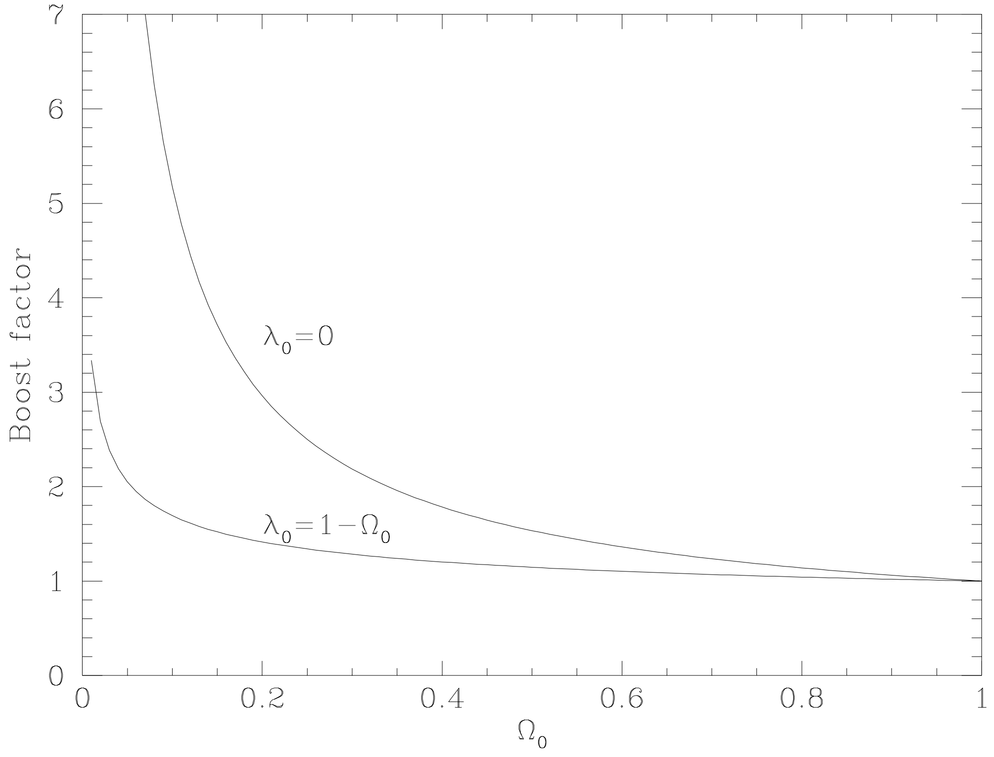

Figure 1: The boost factor.

The boost factor $B(\Omega_0, \lambda_0)$ is plotted as a function of Ω_0 for two classes of cosmologies. The upper curve corresponds to a standard open universe, *i.e.* $\lambda_0 = 0$, whereas the lower curve corresponds to flat universes with $\lambda_0 = 1 - \Omega_0$.

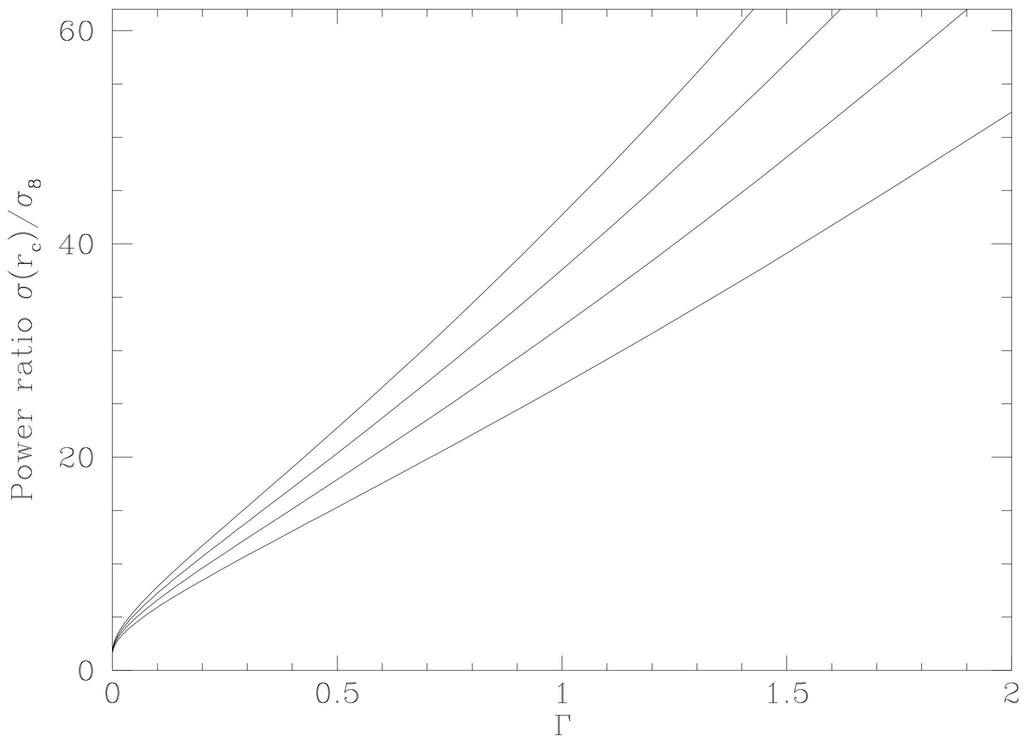

Figure 2: The amplitude ratio.

The ratio of the fluctuation amplitude on the small scale r_c to that at $8h^{-1}$Mpc is plotted as a function of the shape parameter Γ. From top to bottom, the four curves correspond to scales of $3.5h^{-1}$kpc, $7.5h^{-1}$kpc, $16h^{-1}$kpc and $35h^{-1}$kpc, respectively. For $h = 0.5$ and $\Omega_0 = 1$, these four length scales correspond to the masses $10^5 M_\odot$, $10^6 M_\odot$, $10^7 M_\odot$ and $10^8 M_\odot$. The weak additional dependence on Ω_0/h that would result from holding M_c rather than r_c fixed is clearly negligible, since as can be seen, M_c must vary by an entire order of magnitude to offset a mere 20% change in Γ.

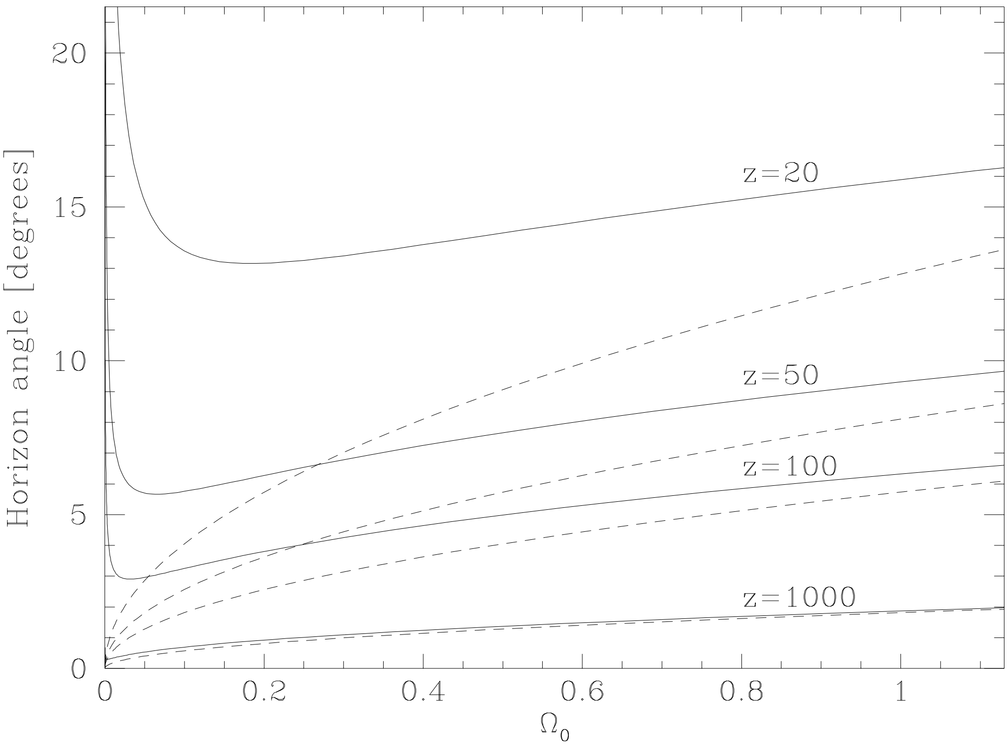

Figure 3: The horizon angle.

The angle in the sky subtended by a horizon volume at redshift z is plotted as a function of Ω_0 for the case with no cosmological constant. The solid lines are the exact results for the four redshifts indicated, and the dashed lines are the corresponding fits using the simplistic approximation $\theta \approx (\Omega_0/z)^{1/2}$.

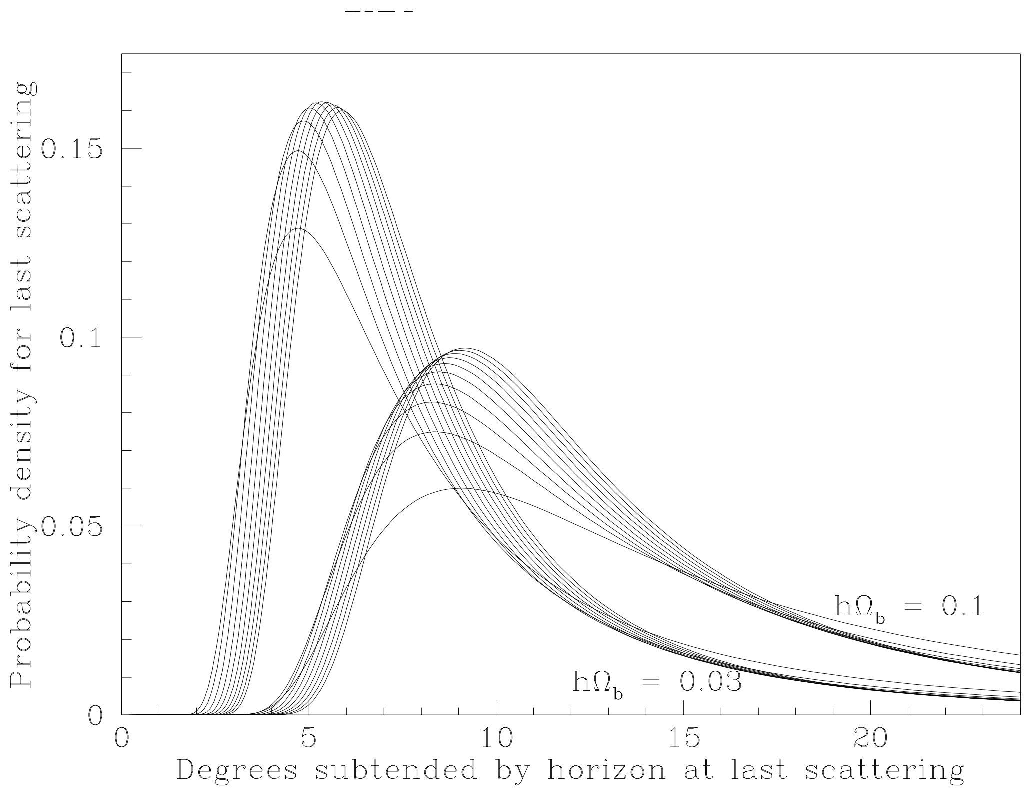

Figure 4: Visibility functions.

The angular visibility function for a fully ionized universe is plotted for different values of Ω_0 and diffuse baryon content $h\Omega_b$. The left group of curves corresponds to $h\Omega_b = 0.03$ and the right to $h\Omega_b = 0.1$. Within each group, from left to right starting at the lowest peak, Ω_0 =0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0.

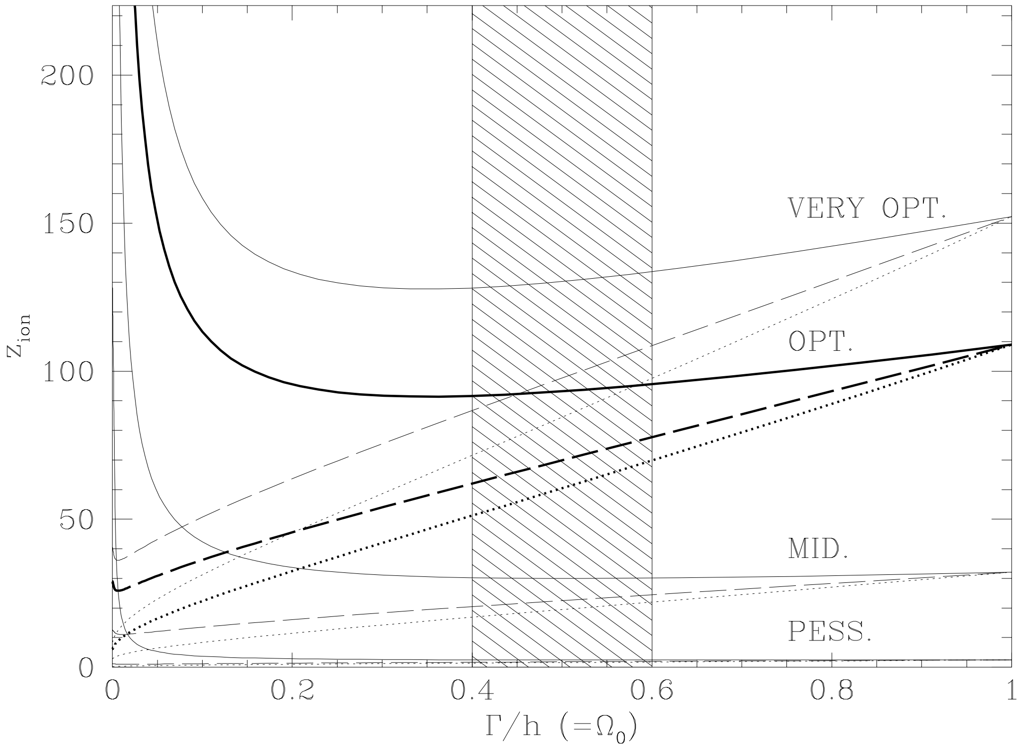

Figure 5: The ionization redshift.

The ionization redshift is plotted for the four scenarios described in Table 1, the heavy lines corresponding to the optimistic scenario. The solid lines are for the open case where $\lambda_0 = 0$. The dashed lines correspond to the flat case where $\lambda_0 = 1 - \Omega_0$. The dotted lines refer to the τCDM model, where $\lambda_0 = 0$ and $\Omega_0 = 1$. Note that the combination Γ/h is really an h-independent quantity: for the open and flat cases, it is simply equal to Ω_0, and for the τCDM case it depends only on g_*, the number of relativistic particle species. The vertical shaded region corresponds to values of Γ preferred by power spectrum measurements, $0.2 < \Gamma < 0.3$, when $h = 0.5$.

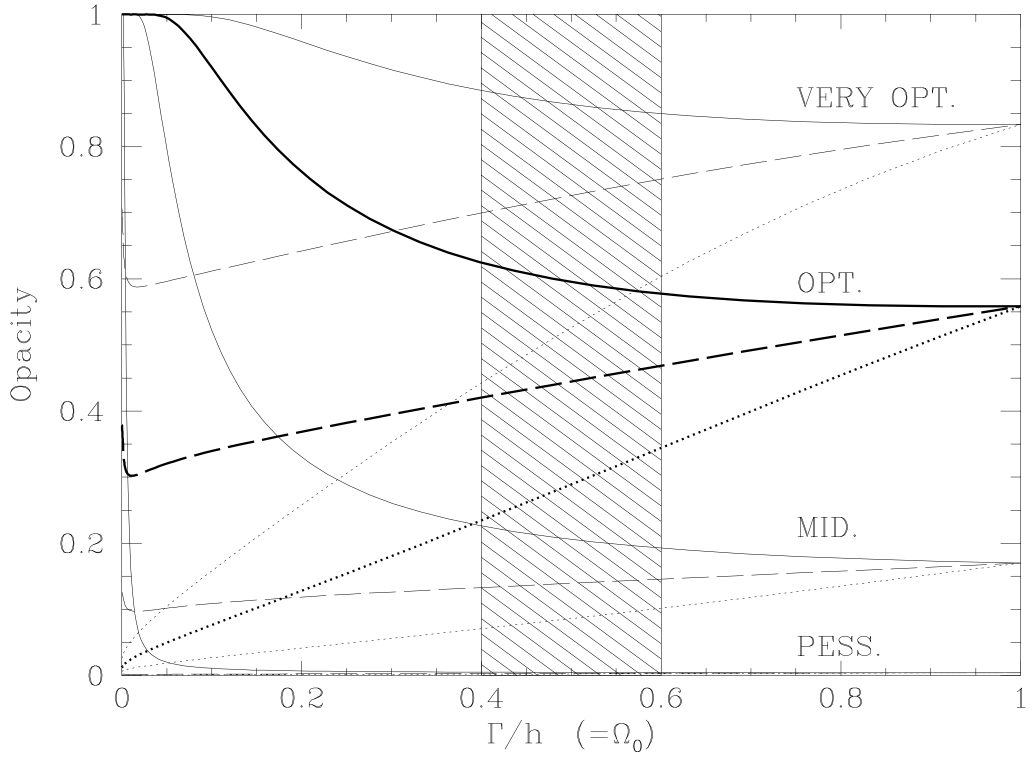

Figure 6: The opacity.

The opacity, the probability that a CBR photon is Thomson scattered at least once since the standard recombination epoch, is plotted for the four scenarios described in Table 1. The solid lines correspond to the open case where $\lambda_0 = 0$. The dashed lines are for the flat case where $\lambda_0 = 1 - \Omega_0$. The dotted lines correspond to the τCDM model, where $\lambda_0 = 0$ and $\Omega_0 = 1$. Just as in Figure 5, note that the combination Γ/h is really an h-independent quantity: for the open and flat cases, it is simply equal to Ω_0, and for the τCDM case it depends only on γ, the number of relativistic particle species. The vertical shaded region corresponds to values of Γ preferred by power spectrum measurements, $0.2 < \Gamma < 0.3$, when $h = 0.5$.

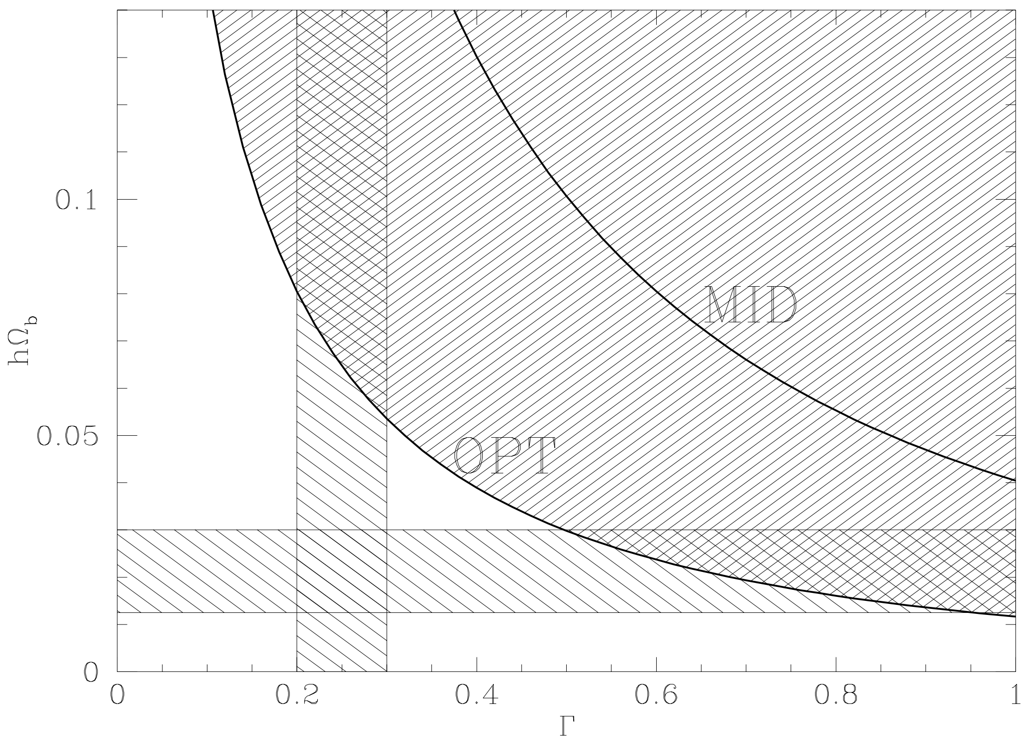

Figure 7: Reionization in τCDM.

The two curves show the baryon density required for 50% opacity in τCDM, *i.e.* for reionization to rescatter 50% of the CBR photons. The upper and lower heavy curves correspond to the middle-of-the road and optimistic scenarios, respectively. Thus even in the optimistic scenario, 50% opacity cannot be obtained outside of the fine-hatched region. The horizontal shaded region corresponds to the values of $h\Omega_b$ allowed by standard nucleosynthesis ($0.01 < h^2\Omega_b < 0.015$) in conjunction with the constraint $0.5 < h < 0.8$. The vertical shaded region corresponds to values of the "shape parameter" Γ preferred by power spectrum measurements. Note that these three regions do not intersect.