Beyond Cosmological Parameters

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Abstract. Observational constraints on spacetime from supernovae Ia, CMB, lensing, Lyman α Forest and galaxy clustering are reviewed, focusing on how the underlying physics (dark matter, dark energy, gravity) can be tested rather than assumed. This is possible because we are simply measuring spacetime, and to 1st order, all observations probe just three cosmological functions, not a dozen dubious cosmological parameters: the cosmic expansion history $H(z)$, the primordial power spectrum $P*(k)$ and the fluctuation growth $g(z,k)$. Measuring these functions will clarify the nature of dark energy (from how its density depends on time), the nature of dark matter (from its power spectrum growth) and the nature of early universe (from the primordial power spectrum).

INTRODUCTION

Traditionally, space was merely a three-dimensional static stage where the cosmic drama played out over time. Einstein's theory of general relativity [1] replaced this by four-dimensional spacetime, a dynamic geometric entity with a life of its own, capable of expanding, fluctuating and even curving into black holes. Now the focus of research is increasingly shifting from the cosmic actors to the stage itself. Triggered by progress in detector, space and computer technology, an avalanche of astronomical data is revolutionizing our ability to measure the spacetime we inhabit on scales ranging from the cosmic horizon down to the event horizons of suspected black holes, using photons and astronomical objects as test particles. The goal of this article is to review these measurements and future prospects, focusing on three key issues:

1. The global topology and curvature of space
2. The expansion history of spacetime and evidence for dark energy
3. The fluctuation history of spacetime and evidence for dark matter

In the process, I will combine constraints from the cosmic microwave background [2], gravitational lensing, supernovae Ia, large-scale structure (LSS), and the hydrogen Lyman α forest[3]. Although it is fashionable to use cosmological data to measure a dozen free "cosmological parameters", I will argue that improved data allow raising the ambition level beyond this, testing rather than assuming the underlying physics. I will discuss how with a minimum of assumptions, one can measure key properties of spacetime itself in terms of a few cosmological functions: the expansion history of the universe, the spacetime fluctuation spectrum and its growth.

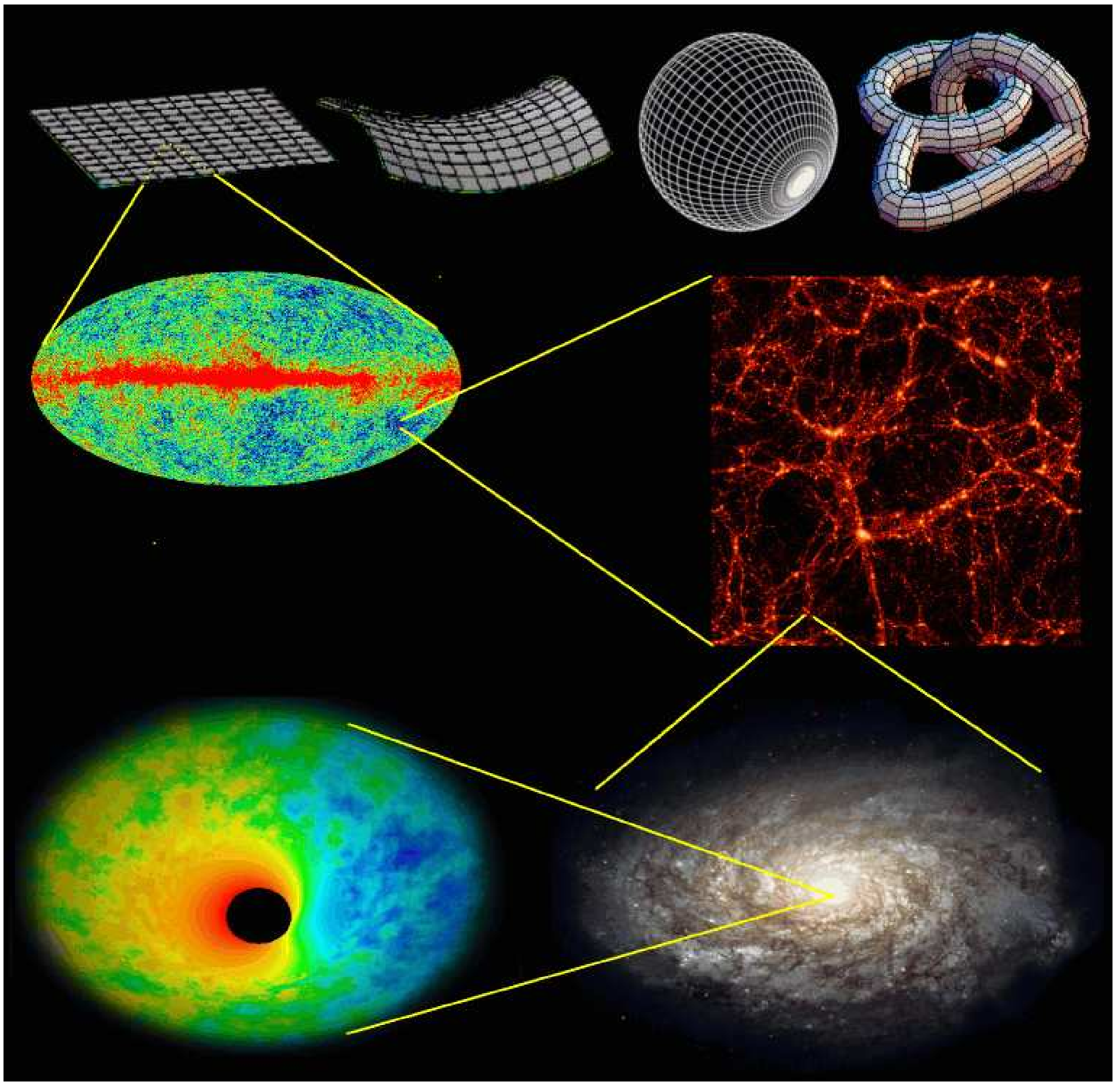

FIGURE 1. Summary of the spacetime issues discussed in this article. One can use photons and astronomical objects as test particles to measure spacetime over 22 orders of magnitude in scale, ranging from the cosmic horizon (probing the global topology of and curvature of space — top) down to galaxies (giving evidence for dark matter), galactic nuclei and binary stellar systems (giving evidence for black holes). The figure illustrates how spacetime ripples at the 10^{-5} level will be imaged by the cosmic microwave background satellite MAP [4] and has grown via gravitational instability into cosmic large-scale structure [5], galaxies and, it seems, black holes [6].

Goals and tools

Before embarking on a survey of spacetime, a brief review of what it is we want to measure and the basic tools at our disposal [7, 8] is in order, as well as a general picture of how spacetime related to the structure of the universe. According to general relativity, spacetime is what mathematicians call a manifold, characterized by a topology and a metric. The topology gives the global structure (Fig. 1, top): is space infinite in all directions like in high-school geometry or multiply connected like say a hypersphere or doughnut so that traveling in a straight line could in principle bring you back home — from the other direction? The metric determines the local shape of spacetime, *i.e.*, the distances and time intervals we measure, and is mathematically specified by a 4×4 matrix at each point in spacetime.

General relativity theory (GR) consists of two parts, each providing a tool for measuring the metric. The first part of GR states that in the absence of non-gravitational forces, test particles (objects not heavy enough to have a noticeable effect on the metric) move along geodesics in spacetime, generalized straight lines, so the observed motions of photons and astronomical objects allow the metric to be reconstructed. I will refer to this as *geometric* measurements of the metric. The second part of GR states that the curvature of spacetime (expressions involving the metric and its first two derivatives) is related to its matter content — in most cosmological situations simply the density and pressure, but sometimes also bulk motions and stress energy. I will refer to such measurements of the metric as *indirect*, because they assume the validity of the Einstein field equations (EFE) of GR.

The broad brush picture

The current consensus in the cosmological community is that spacetime is extremely smooth, homogeneous and isotropic (translationally and rotationally invariant) on large ($\sim 10^{23}\mathrm{m}-10^{26}\mathrm{m}$) scales, with small fluctuations that have grown over time to form objects like galaxies and stars on smaller scales. Cosmic microwave Background (CMB) observations [2] have shown that space is almost isotropic on the scale of our cosmic horizon ($\sim 10^{26}\mathrm{m}$), with the metric fluctuating by only about one part in 10^5 from one direction to another, and combining this with the so-called cosmological principle, the assumption that there is nothing special about our vantage point, implies that space is homogeneous as well. Three-dimensional maps of the galaxy and quasar distribution give more direct evidence for large-scale homogeneity [9].

The fact that the CMB fluctuations are so small is useful, because it allows the intimidating nonlinear partial differential equations governing spacetime and its matter content to be accurately solved using a perturbation expansion. To zeroth order (ignoring the fluctuations), this fixes the global metric to be of the so-called Friedman-Robertson-Walker (FRW) form, which is completely specified except for a curvature parameter and a free function giving its expansion history. To first order, density perturbations grow due to gravitational instability and gravitational waves propagate through the FRW background spacetime, all governed by *linear* equations. Only on smaller scales ($\lesssim 10^{23}\mathrm{m}$) do the fluctuations get large enough that nonlinear dynamics becomes important — in the realm of galaxies, stars and, perhaps, black holes. We will limit our discussion to what can be learned from the 0th and 1st order measurements, covered in turn in the following two sections.

OVERALL SHAPE OF SPACETIME

Curvature of space

The question of whether space is infinite was answered last year with a resounding *maybe*. For a FRW metric, answering this question is equivalent to measuring the curvature of space Fig. 1, specifically the *radius of curvature R*. R is the radius of the hypersphere if space is finite, $R = \infty$ if space is flat, and R is an imaginary number($R^2 < 0$) for saddle-like curvature. Because the three angles of a triangle will add up to 180° in flat space, more if $R^2 > 0$ (like on a sphere) and less if $R^2 < 0$ (like on a saddle) cosmologists have measured R using the largest triangle available: one with us at one corner and the other two corners on the hot opaque surface of ionized hydrogen that delimits the visible universe and emits the CMB, merely 400,000 years after the big bang. Photographs of this surface reveal hot and cold spots of a characteristic angular size that can be predicted theoretically. This characteristic spot size (or, more rigorously, the first peak in the CMB power spectrum [10]) subtends about 0.5° — like the Moon — if space is flat. Sphere-like curvature would make all angles appear larger, so characteristic spots much larger than the Moon would indicate a finite universe curving back on itself, whereas smaller spots would indicate infinite space with negative curvature. Recent experiments have observed the first peak and hints of additional smaller scale peaks [2].

The universe may be infinite, because the measured characteristic spot size is so close to 0.5° that we still cannot tell whether space is perfectly flat or very slightly curved either way. The sharpest current limits on R, obtained by combining all CMB experiments with galaxy clustering data [12, 13] to constrain other parameters, are $|R| > 20h^{-1}\mathrm{Gpc} \approx 10^{27}\mathrm{m}$. This is in sharp contrast to a few years ago, when the most popular models had negatively curved space with $|R| \approx 4h^{-1}$ Gpc. In other words, space now seems to be either infinite or much larger than the observable universe, whose radius is about $9h^{-1}$ Gpc. In 1900, Karl Schwarzschild discussed the possibility that space was curved and published a paper with a lower limit $R > 2500$ light-years $\approx 2 \times 10^{19}$m [14]. A century later, we thus know that the universe is at least another 40 million times larger!

Topology of space

Even if space turns out to be negatively curved or perfectly flat, it might be finite. General relativity does not prescribe the global topology, so various possibilities are possible (Fig. 1, top). The simplest non-trivial model has flat space and the topology of a three-dimensional torus, where opposing faces of a cube of size $L \times L \times L$ are identified to be one and the same. Living in such a universe would be indistinguishable from living in a perfectly periodic one: if $L = 10$ m, you could see the back of your own head 10m away, and additional copies at 20 m, 30 m, and so on — searches for multiple images of cosmological objects have constrained such models. Also, just as a finite guitar string has a fundamental tone and overtones, linear spacetime fluctuations in such a toroidal universe could have only certain discrete wavenumbers. As a result, its CMB power

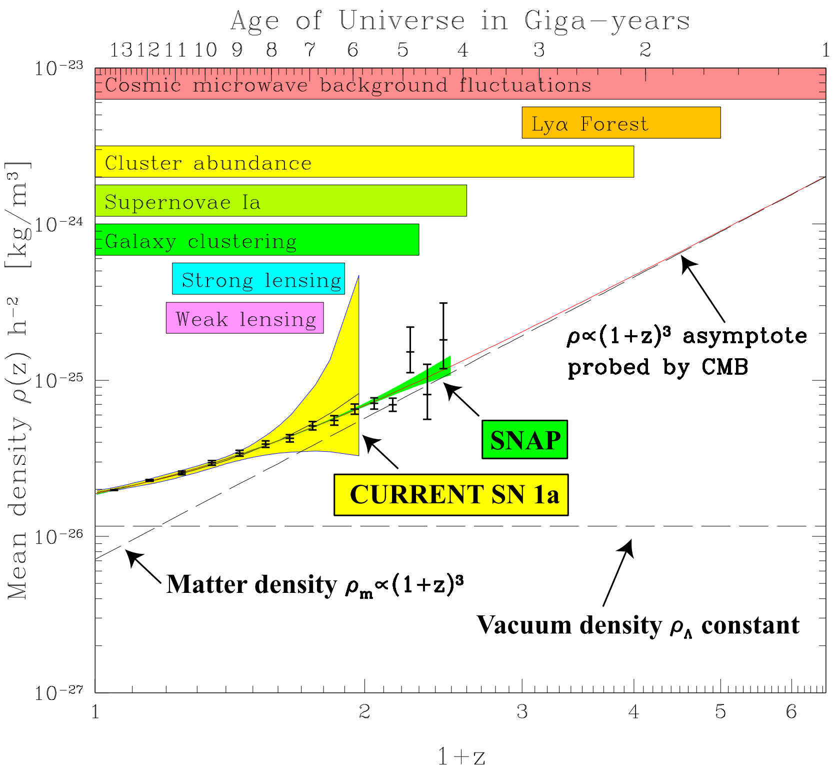

FIGURE 2. Solid curve shows the concordance model [12] for the evolution of the cosmic mean density $\rho(z) \propto H(z)^2$. This curve uniquely characterizes the spacetime expansion history. The horizontal bars indicate the rough redshift ranges over which the various cosmological probes discussed are expected to constrain this function. Because the redshift scalings of all density contributions except that of dark energy are believed to be straight lines with known slopes in this plot (power laws), combining into a simple quartic polynomial, an estimate of the dark energy density $\rho_X(z)$ can be readily extracted from this curve. Specifically, $\rho \propto (1+z)^4$ for the cosmic microwave background (CMB), $\rho \propto (1+z)^3$ for baryons and cold dark matter, $\rho \propto (1+z)^2$ for spatial curvature, $\rho \propto (1+z)^0$ for a cosmological constant and $\rho \propto (1+z)^{3(1+w)}$ for dark energy with a constant equation of state w. Measurement errors are for current SN Ia constraints (yellow band) and a forecast for what the SNAP satellite [19] can do (green band), assuming flat space as favored by the CMB. Error bars are for a non-parametric reconstruction with SNAP.

spectrum would differ on large scales, and it was shown that if the universe were such a torus, then L must be at least of the order of the cosmic horizon [15, 16]. Indeed, it was shown that all three dimensions of the torus must at least about this large to explain the absence of a type of approximate reflection symmetry in the CMB sky [17]. Cosmic topology is now a burgeoning field of study[18], but all available data so far is still consistent with the simplest possible three-dimensional space, the infinite flat Euclidean space that we learned about in high school. The global structure of our four-dimensional spacetime also depends on the beginning and end of time, to which we now turn.

SPACETIME EXPANSION HISTORY

One of the key quantities that cosmologists yearn to measure is the function $a(t)$, describing the expansion of the universe over time — if space is curved, a is simply the magnitude of the radius of curvature, $a = |R|$. A mathematically equivalent function more closely related to observations is the Hubble parameter as a function of redshift, $H(z)$, giving the cosmic expansion rate and defined by $H \equiv \frac{d}{dt}\ln a$, $1+z \equiv a(t_{\rm now})/a(t)$.

What $\rho(z)$ tells us about dark energy

Squaring our curve $H(z)$ gives us the cosmic matter density (Fig. 2). If the Einstein Field equations of GR are correct, then the mean density of the universe is given by the Friedmann equation

$$\rho(z) = \frac{3H(z)^2}{8\pi G}. \tag{1}$$

Here G is Newton's gravitational constant and, if space is curved, the density ρ is defined to include an optional curvature contribution $\rho_{\rm curv} \equiv -3c^2/8\pi GR^2$, where c is the speed of light. Conveniently, all standard components of the cosmic matter budget contribute simple straight lines to this plot, because their densities drop as various power laws as the universe expands. For instance, the densities of both ordinary and cold dark matter particles are inversely proportional to the volume of space, scaling as $\rho \propto (1+z)^3$.

The cosmic density $\rho(z)$ measured from SN Ia and CMB (yellow band in Fig. 2) was higher in the past, but rises slower than $(1+z)^3$ towards higher z, with a shallower slope than 3 at recent times. Fig. 2 shows that although the cosmic density $\rho(z)$ measured from SN Ia and CMB was indeed higher in the past, the curve rises slower than this towards higher redshift, with a shallower slope than 3 at recent times. This is evidence for the existence of *dark energy*, a substance whose density does not rise rapidly with z. Adding a cosmological constant contribution $\rho_\Lambda \approx 4\times 10^{-26}$ kg/m^3 (about 2/3 of the current matter budget) whose density is, by definition, constant, provides a good fit to the measurements (Fig. 2). This 1998 discovery [20, 21] stunned the scientific community and triggered a worldwide effort to determine the nature of the dark energy. A model-independent approach involves measuring the curve $\rho(z)$ more accurately with (Fig. 2), to determine whether independent measurements of $\rho(z)$ agree, so that we can rule out problems with observations, and to determine the time-dependence of dark energy density $\rho_X(z)$. If it is constant, we may have measured vacuum energy/Einstein's cosmological constant, and if not, we should learn interesting physics about a new scalar quintessence field, or whatever is responsible. A less ambitious approach that is currently popular is assuming that the equation of state (pressure-to-density ratio) w of the dark energy is constant [22, 23], which is equivalent to assuming that $\rho_X(z)$ is a straight line in Fig. 2 with a free amplitude and slope.

What $\rho(z)$ tells us about our origin and destiny

If we can understand the different components of the cosmic matter budget well enough to extrapolate the curve $\rho(z)$ from Fig. 2 to the distant past and future, we can use the Friedmann equation to solve for $a(t)$ and obtain information about the origin and ultimate fate of spacetime. $a(t) = 0$ in the past or future would correspond to a singular big bang or big crunch, respectively, with infinite density $\rho(z)$. As to the past, such extrapolation seems justified at least back to the first second after the big bang, given the success of big bang nucleosynthesis in accounting for the primordial light element abundances [24]. Regarding the very beginning, the jury is still out. Extrapolation back to the very beginning is more speculative. According to the currently most popular scenario, a large and nearly constant value of ρ at $t \lesssim 10^{-34}$ seconds caused exponential expansion $a(t) \propto e^{Ht}$ during a period known as inflation [25], successfully predicting both negligible spatial curvature and a nearly scale-invariant adiabatic scalar power spectrum[10] with subdominant gravitational waves. A rival ekpyrotic model inspired by string theory and a related eternally oscillating model have attracted recent attention[26, 27, 28]. If the density approaches the Planck density ($10^{97}\,\mathrm{kg/m^3}$) as $t \to 0$, quantum gravity effects for which we lack a fundamental theory should be important, and a host of speculative scenarios have been put forward for what happened at $t \sim 10^{-43}$ seconds. A very incomplete sample includes the Hawking-Hartle no-boundary condition [29], God creating the universe, the universe creating itself [30], and so-called pre-big-bang models [31]. Another possibility is that the Planck density was never attained and that there was no beginning, just an eternal fractal mess of replicating inflating bubbles, with our observed spacetime being merely one in an infinite ensemble of regions where inflation has stopped [32, 33].

As to the future, the expansion can only stop ($H = 0$) if the effective density $\rho(z)$ drops to zero. The only two density contributions that can in principle be negative are those of curvature (measured to be negligible) and dark energy (measured to be positive), suggesting that the universe will keep expanding forever. Indeed, if the dark energy density stays constant, we are now entering another inflationary phase of exponential expansion ($a(t) \propto e^{Ht}$), and in about 10^{11} years, our observable universe will be dark and lonely with almost all extragalactic objects having disappeared across our cosmic horizon [34]. Hovever, such conclusions must clearly be taken with a grain of salt until the nature of dark energy is understood.

How to measure $\rho(z)$

In conjunction with the R, the curve $H(z)$ can be measured geometrically, using photons as test particles. Objects of known luminosity (called standard candles) or known physical size (called standard yardsticks) at different redshifts can be used to determine $H(z)$ by comparing their measured brightness or angular size with theoretical predictions, which follow from computing the trajectories of nearly parallel light rays and depend only on $H(z)$ and the (apparently negligible) curvature of space [35, 36]. The best standard candles to date are supernovae of type Ia, and 92 SN Ia [20, 21] were used [36] to measure $H(z)$ and thereby $\rho(z)$ (Fig. 2). The best standard yardstick so far is the characteristic CMB spot size, suggesting that space is flat.

$H(z)$ can also be measured indirectly. As discussed below, $H(z)$ affects the growth of density fluctuations and can therefore be probed by galaxy clustering and other techniques (Fig. 2). Such fluctuation measures have constrained matter to make up no more than about a third of the critical density needed to explain why space is flat. This Enron-like accounting situation provides supernova-independent evidence for dark energy [12, 13, 39].

GROWTH OF COSMIC STRUCTURE

While SN Ia and CMB peak locations have recently revolutionized our knowledge of the metric to 0th order (curvature, topology and expansion history), other observations are probing its 1st order fluctuations with unprecedented accuracy. These perturbations come in two important types. The first are gravitational waves, hitherto undetected ripples in spacetime which propagate at the speed of light without growing in amplitude. The second are density fluctuations, which can get amplified by gravitational instability (Fig. 1) and are being measured by CMB, gravitational lensing and the clustering of extragalactic objects, notably galaxies and gas clouds absorbing quasar light (the so-called Lyman α forest, LyαF) over a range of scales and redshifts (Fig. 3).

Plane wave perturbations of different wavenumber evolve independently by linearity, and are so far consistent with having uncorrelated Gaussian-distributed amplitudes as predicted by most inflation models [25]. The 1st order density perturbations are therefore characterized by a single function $P(k,z)$, the *power spectrum* [10], which gives the variance of the fluctuations as a function of z and wavenumber k. $P(k,z)$ depends on and can therefore teach us about three things: The cosmic matter budget, the seed fluctuations created in the early universe and Galaxy formation. A challenge is to robustly disentangle the three. We are not there yet, but new data is making this increasingly feasible because each of the probes in Fig. 3 involve different physics and is affected by the three in different ways as outlined below.

Given the profusion of recent measurements of $H(z)$ and $P(k,z)$, it is striking that there is a fairly simple model that currently seems to fit everything (Fig. 2 and Fig. 4). In this so-called concordance model [12, 13, 39, 40], the cosmic matter budget consists of about 5% ordinary matter (baryons), about 30% cold dark matter, about 0.1% hot dark matter (neutrinos) and about 65% dark energy based on CMB and LSS observations, in consistent with LyαF [41], lensing [42, 43] and SN Ia [20, 21]. The seed fluctuations created in the early universe are consistent with the inflation prediction of a simple power law $P(k,z) \propto k^n$ early on, with $n = 0.9 \pm 0.1$ [12, 13]. Galaxy formation appears to have heated and reionized the universe not too long before $z = 6$ based on the LyαF [44].

Although inferences about things like the expansion history, the matter budget and the early universe involve many assumptions (about the nature of dark energy, dark matter, gravity, galaxy formation, and so on), the avalanche of new cosmology data allows raising the ambition level and testing these assumptions about the underlying physics. Given the matter budget and the expansion history $H(z)$, theory predicts the complete

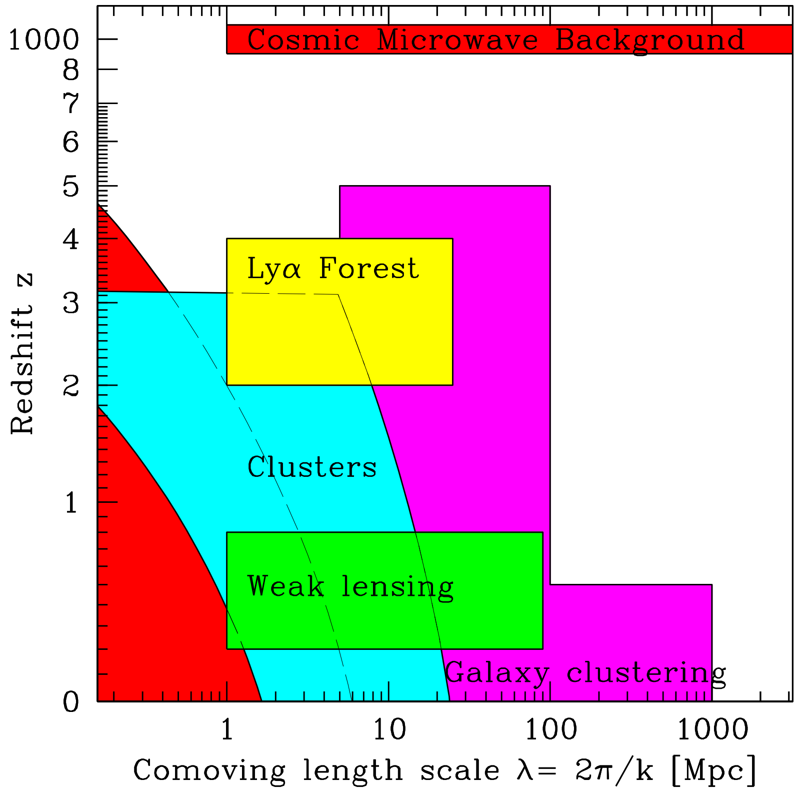

FIGURE 3. Shaded regions show ranges of scale and redshift over which various observations are likely to measure spacetime fluctuations over the next few years. The lower left region, delimited by the dashed line, is the non-linear regime where rms density fluctuations exceed unity in the "concordance" model from [12].

time-evolution of linear clustering, so measuring its redshift dependence (Fig. 3) offers redundancy and powerful cross-checks.

Many power spectrum probes are improving rapidly (Fig. 3). **Gravitational lensing** uses photons from distant galaxies as test particles to measure the metric fluctuations caused by intervening matter, as manifested by distorted images of distant objects, and first measured $P(k,z)$ in 2000. Three-dimensional mapping of the universe with **galaxy redshift surveys** offers another window on the cosmic matter distribution, through its gravitational effects on galaxy clustering. This field is currently being transformed by the 2 degree Field (2dF) survey and the Sloan Digital Sky Survey, and complementary surveys will map high redshifts and the evolution of clustering. The abundance of **galaxy clusters** at different epochs, as probed by optical, x-ray, CMB or gravitational lensing surveys, is a sensitive probe of $P(k,z)$ on smaller scales and the **LyαF** offers a new and exciting probe of matter clustering on still smaller scales when the universe was merely 10-20% of its present age. **CMB** experiments probe $P(k,z)$ through a variety of effects as far back as to $z > 10^3$, with exciting new fronts being increased sensitivity [4], increased angular resolution and CMB polarization (still undetected).

These complementary probes can be combined to break each others' degeneracies and

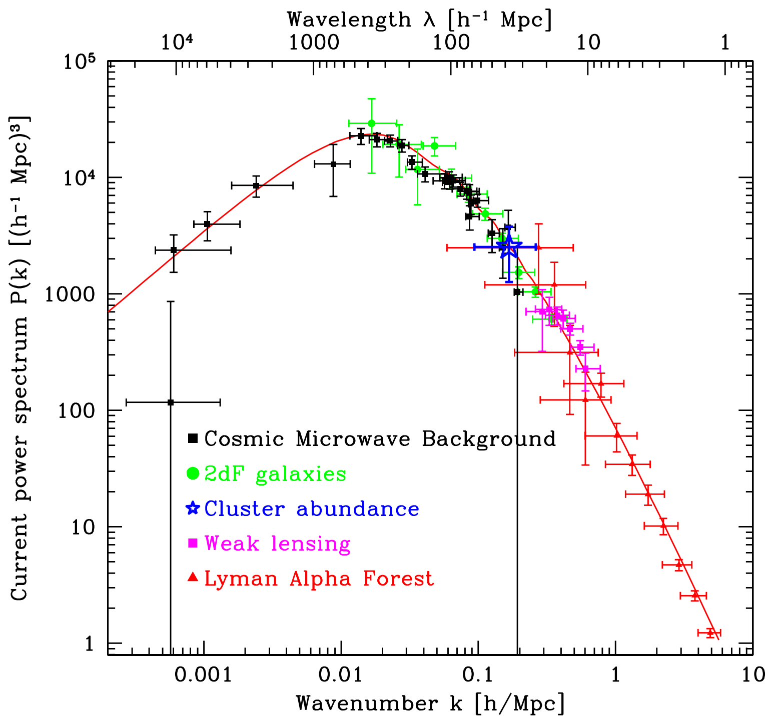

FIGURE 4. Measurements of the current ($z = 0$) power spectrum of density fluctuations computed as described in [48], assuming the matter budget of [13] and reionization at $z = 8$. The CMB measurements combine the information from all experiments to date as in [48]. LSS points are from a recent analysis [45] of the 3D distribution of 2dF galaxies [9], and correcting them for bias shifts them vertically ($b = 1.3$ assumed here) and should perhaps blue-tilt them slightly. The cluster error bars reflect the spread in the literature. The lensing points are based on [49]. The LyαF points are from a reanalysis [50] of [41] and have an overall calibration uncertainty around 17%. The curve shows the concordance model of [13].

independently measure the matter budget, the primordial power spectrum and galaxy formation details [47]. The power spectra measured by CMB, LSS, lensing and LyαF are the product of the three terms: the primordial power spectrum, a so-called transfer function quantifying the subsequent fluctuation growth, and, for LSS and LyαF only, a so called bias factor accounting for the fact that the measured galaxies and gas clouds may cluster differently than the underlying matter [11].

Disentangling bias and systematic errors: Galaxy bias has now been measured from data and found to be of order unity for typical 2dF galaxies [40, 46], and LyαF bias may be computable with hydrodynamics simulations [41, 50]. Although CMB, LSS, lensing and LyαF each comes with caveats of their own, their substantial overlap (Fig. 3) should allow disagreements between data sets to be distinguished from disagreements between data and theory.

Disentangling primordial power from the matter budget: The transfer function can be disentangled from the primordial power since it depends on the matter budget, and conveniently in rather opposite ways for CMB than for lower z $P(k)$ measurements (LSS, lensing, LyαF). For instance, increasing the cold dark matter density $h^2\Omega_c$ shifts the galaxy power spectrum up to the right and the CMB peaks down to the left if the primordial spectrum is held fixed. Adding more baryons boosts the odd-numbered CMB peaks but suppresses the galaxy power spectrum rightward of its peak and also makes it wigglier. Increasing the dark matter percentage that is hot (neutrinos) suppresses small-scale galaxy power while leaving the CMB almost unchanged. This means that combining CMB with other data allows unambiguous determination of the matter budget, and the primordial power spectrum can then be inferred. Combining CMB temperature and polarization measurements also helps in this regard, because the characteristic wiggles imprinted by the baryons and dark matter are out of phase for the two, whereas wiggles due to the primordial spectrum would of course line up for the two [48].

Although the best is still to come in this area, the basic conclusion that the universe is awash in nonbaryonic dark matter is already supported independently by CMB, LyαF, galaxy surveys, cluster counting and lensing. The agreement on the baryon density between fluctuation studies (CMB + galaxy surveys) and nucleosynthesis and on the dark energy density between fluctuation studies and SN Ia are both indications that spacetime fluctuation measurements are on the right track and will live up to their promise in this decade of precision cosmology.

Constraints on dark matter from nonlinear clustering

On small scales, the linear perturbation expansion eventually breaks down as density fluctuations grow to be of order unity, collapsing to form a variety of interesting astrophysical objects. Although the theoretical predictions are more difficult in this regime, the metric can still be accurately measured using photons and astrophysical objects as test particles. The gravitational potential well is probed by strong gravitational lensing of photons through its distorting effect on background objects [51] and also by the motions of massive objects like galaxies, stars or gas clouds. The orbital parameters in a binary system reveal the masses of the two objects, just as we once weighed the Sun exploiting Earth's orbit around it. In more complicated systems, the central mass distribution can be inferred statistically from velocity dispersions observed in the vicinity. These basic tools have revealed surprises on three vastly different scales: dark matter in galaxies and clusters ($\sim 10^{20-23}$m), supermassive black holes in galactic bulges ($\sim 10^{10} - 10^{13}$m) and stellar-mass black holes ($\sim 10^4 - 10^5$m). Recent black hole reviews include [52, 53, 54].

As noted by Zwicky in 1933 [55], the amount of mass in galaxies and galaxy clusters inferred from rotation curves or velocity dispersions exceeds the mass of luminous matter by a large factor. Precision measurements with a variety of techniques have confirmed this finding, providing evidence that both galaxies and clusters are accompanied by roughly spherical halos of cold dark matter. This dark matter evidence is independent of that from linear perturbation theory described above, yet produces roughly consistent estimates of the total cosmic dark matter density [56, 57].

New measurements such as mapping tidal streamers, stripy remnants of galaxies cannibalized by the Milky Way in the past, may allow 3D reconstruction of our own dark matter halo, and early results suggest that it may be elliptical rather than perfectly spherical [58]. Measurements of the shape and substructure of dark matter halos can probe the detailed nature of the dark matter. Indeed, computer simulations with cold dark matter composed of weakly interacting particles appear to predict overly dense cores in the centers of galaxies and clusters, and that there should be about 10^3 discrete dark matter halos in our Galactic neighborhood (the Local Group), in contrast to the less than 10^2 galaxies actually observed. These halo profile and substructure problems have triggered talk of a cold dark matter crisis and much recent interest in self-interacting dark matter [59], warm dark matter [60] and other more complicated dark matter models which suppress cores and substructure. It is not obvious that there really is a crisis, because baryonic feedback properties may be able to reconcile vanilla cold dark matter with observations and substantial halo substructure has recently been detected with gravitational lensing [61], but this active research area should teach us more about dark matter properties whatever they turn out to be.

OUTLOOK

I have surveyed recent measurements of spacetime over a factor of 10^{22} in scale, ranging from the cosmic horizon down to the event horizon of black holes. On the largest scales, evidence supports a flat infinite space and eternal future time. The growth of spacetime fluctuations has suggested that about 30% of the cosmic matter budget is made up of (mostly cold) dark matter, about 5% ordinary matter and the remainder dark energy. There is further evidence for the same dark matter in the halos of galaxies and clusters. Finally, spacetime seems to be full of black holes, both supermassive ones in the centers of most galaxies and stellar mass ones wherever high mass stars have died [52, 53, 54].

The Devil's advocate requires that we clarify the assumptions underlying these conclusions. The geometric test particle observations have measured the spacetime metric, but all inferences about dark energy, dark matter and the inner parts of black holes assume that the Einstein Field Equations (EFE) of GR are valid. Attempts have been made to explain away all three by modifying the EFE. So-called scalar-tensor gravity has been found capable of giving accelerated cosmic expansion without dark energy [63]. Although not an ab initio theory, the approach known as modified Newtonian dynamics (MOND) attempts to explain galaxy rotation curves without dark matter [64]. It is not inconceivable that the EFE can be modified to avoid black hole singularities [66].

So could dark energy, dark matter and black holes be merely a modern form of epicycles, which just like those of Ptolemy can be eliminated by modifying the laws of gravity [36, 65, 67, 68]? The way to answer this question is to test the EFE observationally, by embedding them in a larger class of equations and quantifying the observational constraints. So far, the true theory of gravity has been shown to extremely close to GR in the regime probed by solar system dynamics and binary pulsars [7, 8] and the MOND-loophole has been at least partially closed [69, 70, 71]. However, this does *not* imply that the true theory of gravity must be indistinguishable from GR in all contexts, in particular for very compact objects [62] or for cosmology [7, 8], so testing gravity remains a fruitful area of research. Such tests continue even in the laboratory [72], testing the gravitational inverse square law down to millimeter scales to probe possible extra dimensions [73].

In conclusion, the coming decade will be exciting: an avalanche of astrophysical observations are measuring spacetime with unprecedented accuracy, allowing us to test whether it obeys Einstein's field equations, and consequently whether dark energy, dark matter and black holes are for real.

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