REMOVING POINT SOURCES FROM CMB MAPS

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ABSTRACT

For high-precision cosmic microwave background (CMB) experiments, contamination from extragalactic point sources is a major concern. It is therefore useful to be able to detect and discard point source contaminated pixels using the map itself. We show that the sensitivity with which this can be done can often be greatly improved (by factors between 2.5 and 18 for the upcoming Planck mission) by a customized hi-pass filtering that suppresses fluctuations due to CMB and diffuse galactic foregrounds. This means that point source contamination will not severely degrade the cleanest Planck channels unless current source count estimates are off by more than an order of magnitude. A catalog of around 40,000 far infra-red sources at 857 GHz may be a useful by-product of Planck.

1. INTRODUCTION

If future CMB experiments are to produce high-precision measurements of cosmological parameters (Jungman *et al.* 1996; Bond *et al.* 1997; Zaldarriaga *et al.* 1997), they must remove foreground contamination from galactic dust, synchrotron and free-free emission as well as extragalactic point sources with comparable accuracy (see *e.g.* Brandt *et al.* 1994; Tegmark & Efstathiou 1996 – hereafter TE96; Bersanelli *et al.* 1996). Fortunately, foregrounds differ from CMB fluctuations in several ways, all of which can be used as weapons against them, in combination:

1. Their non-Gaussian behavior can be used to discard severely contaminated regions (*e.g.*, bright point sources, the Galactic plane).
2. Their frequency dependence can be used to subtract them out by taking linear combinations of maps at different frequencies.
3. Their power spectra can be compared with the one measured, to fit out remaining foregrounds.

Extragalactic point sources are one of the most menacing foregrounds, for several reasons:

- Their spectral index varies much more than for other foregrounds.
- Many sources exhibit substantial time variability.
- Their abundance is very poorly known over much of the CMB frequency range.

The variation of their frequency dependence in a random way across the sky (from source to source) substantially degrades the effectiveness of weapon 2 (Tegmark 1998), and time-variability of both flux and spectral shape (*e.g.*, Gutierrez de la Cruz *et al.* 1998) further complicates subtraction attempts. As a complement to 2, it is therefore important to make as much use as possible of 1. This is the purpose of the present *Letter*.

If point sources give the only non-Gaussian contribution to a map, then a useful way to implement 1 is to discard all pixels whose temperature lies more than ν standard deviations σ above the mean. Chosing $\nu = 5$ ensures that this will falsely reject only a negligible fraction of order 3×10^{-7} of all uncontaminated pixels. The accuracy with which this method will be able to clean the CMB maps from the upcoming Planck satellite (Bersanelli et al 1996) has recently been estimated by Gawiser & Smoot (1997), Toffolatti et al (1998, hereafter To98) and Guiderdoni *et al.* (1998). Similar estimates have been made for the MAP experiment (Refrigier *et al.* 1998). For such high signal-to-noise experiments, the rms pixel fluctuations σ are not dominated by detector noise but by CMB (or, in some channels, by galactic foregrounds). We will show that much of these fluctuations (which hamper our ability to detect point sources) can be removed by an appropriately chosen band-pass filter, and that consequently, the threshold $\nu\sigma$ at which point sources can be removed can be substantially lowered. We derive our method in §2, apply it to Planck in §3 and discuss our results in §4.

2. METHOD

If there are point sources with fluxes S_i at sky positions given by unit vectors $\widehat{\mathbf{r}}_i$, then the resulting sky temperature x is

$$x(\widehat{\mathbf{r}}) = c\sum_i S_i\delta(\widehat{\mathbf{r}}_i,\widehat{\mathbf{r}}) + \sum_{\ell m} a_{\ell m}Y_{\ell m}(\widehat{\mathbf{r}}), \tag{1}$$

where δ is a Dirac delta function and the spherical harmonic coefficients $a_{\ell m}$ contain the combined contributions from CMB, galactic foregrounds and detector noise. Here c is the conversion factor between brightness and temperature, given by (see, *e.g.*, equation (3) in TE96)

$$c = c_*\frac{\sinh^2\eta/2}{\eta^4},\quad c_*\equiv\frac{2}{k}\left(\frac{hc}{kT_{cmb}}\right)^2\approx\frac{40\,\mathrm{mK}}{\mathrm{MJy/sr}} \tag{2}$$

where $\eta\equiv h\nu/kT_{cmb}\approx\nu/56.8\,\mathrm{GHz}$. The observed map is $x(\widehat{\mathbf{r}})$ convolved with the beam function B. To maximize

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Available in color from h t t p://www.sns.ias.edu/~max/pointsources.html

our sensitivity to the point sources, we wish to convolve it with an additional function W, giving a filtered map $y(\widehat{\mathbf{r}}) \equiv (W \star B \star x)(\widehat{\mathbf{r}})$ ($\star$ denotes convolution). Equation (1) gives

$$y(\widehat{\mathbf{r}}) = c\sum_i S_i (W\star B)(\widehat{\mathbf{r}}_i\cdot\widehat{\mathbf{r}}) + \sum_{\ell m} W_\ell B_\ell a_{\ell m} Y_{\ell m}(\widehat{\mathbf{r}}). \quad (3)$$

Here we have taken both the filter B and the beam W to be spherically symmetric, so that they are given by the coefficients W_ℓ and B_ℓ in a Legendre polynomial expansion:

$$B(\cos\theta) = \sum_\ell \left(\frac{2\ell+1}{4\pi}\right) B_\ell P_\ell(\cos\theta), \quad (4)$$

$$W(\cos\theta) = \sum_\ell \left(\frac{2\ell+1}{4\pi}\right) W_\ell P_\ell(\cos\theta). \quad (5)$$

Equation (3) tells us that if the contribution from overlap of nearby sources is negligible (we will see that this is a good approximation for Planck), then the point source contribution in the direction of the i^{th} source is $y(\widehat{\mathbf{r}}_i) = AS_i$, where the normalization constant $A \equiv (W\star B)(1)c$, since $\widehat{\mathbf{r}}_i \cdot \widehat{\mathbf{r}}_i = 1$. This constant can be re-written as

$$A = (W\star B)(1)c = c\sum_\ell \left(\frac{2\ell+1}{4\pi}\right) B_\ell W_\ell. \quad (6)$$

In other words, equation (3) tells us that the peak brightness of a source in the normalized map $y(\widehat{\mathbf{r}})/A$ gives its strength S in flux units (Janskys). This is true for any choice of our filter W, so we simply wish to chose W so that it minimizes the variance σ^2 in this map. These fluctuations come from the $a_{\ell m}$ in equation (3), *i.e.*, from CMB, pixel noise and galactic foregrounds, all which act as unwanted noise now that point source detection is our objective. Modeling these as Gaussian random fields as in TE96, we have

$$\langle a^*_{\ell m} a_{\ell' m'}\rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell^{tot}, \quad (7)$$

where C_ℓ^{tot} is the sum of the power spectra of the CMB, the noise and various Galactic foreground components. Using this, equation (3) and equation (6), we find that the variance of our point source map $y(\widehat{\mathbf{r}})/A$ is

$$\sigma^2 \equiv V\left[\frac{y(\widehat{\mathbf{r}})}{A}\right] = c^2\frac{\sum_\ell\left(\frac{2\ell+1}{4\pi}\right) B_\ell^2 C_\ell^{tot} W_\ell^2}{\left[\sum_\ell\left(\frac{2\ell+1}{4\pi}\right) B_\ell W_\ell\right]^2}. \quad (8)$$

It is easy to show that this variance is minimized if we chose the filter to be

$$W_\ell \propto \frac{1}{B_\ell C_\ell^{tot}}, \quad (9)$$

which gives

$$\sigma^2 = c^2\left[\sum_\ell\left(\frac{2\ell+1}{4\pi}\right)/C_\ell^{tot}\right]^{-1}. \quad (10)$$

In summary, the outlyer removal method can eliminate all point sources with flux $S > S_c \equiv \nu\sigma$, where σ is given by equation (10).

We note that our assumption of symmetric beams is by no means necessary. Indeed, our filtering method is readily generalized to the case of arbitrary beam shape and arbitrary known profiles of the sources, in a manner analogous to Appendix A of Haehnelt & Tegmark (1996).

3. APPLICATION TO PLANCK

We will now illustrate our method with an application to the upcoming Planck mission, to see by what factor it reduces σ.

3.1. *Assumptions about foregrounds, noise and CMB*

For the Galactic foreground power spectra, we use the model $C_\ell(\nu) \propto c^2 B(\nu)^2 \ell^{-3}$ (TE96; Bersanelli *et al.* 1996) for dust, synchrotron and free-free emission, where B is the sky brightness measured in MJy/sr. We model the frequency dependence as $B(\nu) \propto \nu^{-0.15}$ for free-free emission and as $B(\nu) \propto \nu^{-0.9}$ for synchrotron emission (see Platania *et al.* 1998 and references therein). For dust, we assume $B(\nu) \propto \nu^{3+\beta}/(e^{h\nu/kT}-1)$, with a dust temperature $T = 20$K and an emissivity $\beta = 1.5$ (Kogut *et al.* 1996, hereafter K96).

The combined DIRBE and IRAS dust maps suggest a slightly shallower slope $\ell^{-2.5}$ (Schlegel *et al.* 1998), but a recent analysis of the DIRBE maps has shown no evidence of a departure from an ℓ^{-3} power law (Wright 1998) for $\ell \lesssim 300$, and we will see that only the behavior at low ℓ matters for the present analysis.

We normalize the power spectra based on the DIRBE-DMR cross-correlation analysis of K96, which gives rms fluctuations of 2.7μK for dust and 7.1μK for free-free emission at 53 GHz on the COBE angular scale, in good agreement with the DIRBE cross-correlation results for the Saskatoon map (de Oliveira-Costa *et al.* 1997) and 19 GHz map (de Oliveira-Costa *et al.* 1998). We normalize the synchrotron model to give 11μK at 31 GHz, the K96 upper limit, since cross-correlation between the 408 MHz and 19 GHz emission indicates a value in this range (de Oliveira-Costa *et al.* 1998). For radio and infrared point sources, we use the source count model of To98.

The noise power spectrum is $C_\ell^{noise} = (\mathrm{FWHM}\sigma_n)^2/B_\ell^2$ (Knox *et al.* 1995; TE96), where σ_n is the rms noise in a pixel of area FWHM^2. We assume Gaussian beams of rms width θ, which corresponds to $B_\ell = e^{\theta^2\ell(\ell+1)/2}$, where $\theta = \mathrm{FWHM}/(8\ln 2)^{1/2}$. The values of ν, FWHM and σ_n for the Planck channels are taken from *http://astro.estec.esa.nl/Planck/*.

We compute the CMB power spectrum with CMBFAST (Seljak & Zaldarriaga 1996) for three COBE-normalized models. We use a standard cold dark matter model (SCDM) with $n = 1$, $h = 0.5$, $h^2\Omega_b = 0.015$ as well as two models that agree better with observational data (Wang *et al.* 1998). These are ΛCDM, a flat model with a cosmological constant (as SCDM except that $\Omega_\Lambda = 0.5$), and OCDM, an open model (as SCDM except that $\Omega_\Lambda = 0$, $\Omega = 0.5$, $h = 0.65$).

3.2. *Results*

The optimal filter W for Planck channel 5 in the ΛCDM model is shown in real space in Figure 1 and in the Fourier (multipole) domain in Figure 2. The ΛCDM results for all channels are summarized in Table 1, and it is seen that the sensitivity σ is improved by a factor ranging from 2.5 to 18. This can be qualitatively understood from Figure 2, which shows that W is essentially a band-pass filter, targeting those multipoles where the combined fluctuations from CMB, foregrounds and noise are minimal. The table shows that the dominant sky signal (CMB for channels 1-7, dust for channels 8-10) is suppressed by even larger factors, at the cost of higher detector noise. The filtering technique therefore helps the most for high resolution experiments where the signal-to-noise level per resolution element is substantially greater than unity. This should be attainable for instance for several of the upcoming interferometer experiments; see Smoot (1997) for a recent review. Conversely, the filtering helps only marginally (with gains below a factor of 2) for the upcoming MAP satellite, for which we repeated our analysis using the specifications from *http://map.gsfc.nasa.gov*. This is because the MAP detector noise is not too far below the fluctuation levels from CMB and foregrounds to start with, leaving less room than in the Planck case for filtering to improve the situation. The same conclusion is drawn in the MAP analysis of Refrigier *et al.* (1998).

Table 1 shows that the number of sources removed changes by an even greater factor than σ does. This is because the differential source counts of To98 are quite steep. Channel 10 is especially notable: here Planck should be able to produce a catalog with around 40,000 sources, undoubtedly useful in its own right, as compared to a measly 300 without filtering. Our filtering technique should also be useful for constructing a catalog of SZ-clusters from the 217 GHz map, as described by Aghanim *et al.* (1997).

The SCDM and OCDM models give results quite similar to Table 1, with gain factors differing by less than 20%.

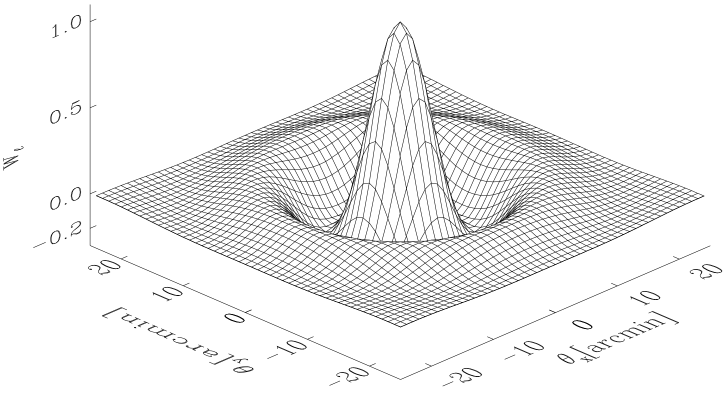

FIG. 1 — The convolution filter W is plotted in real space for channel 5 of Planck.

4. DISCUSSION

We have presented a technique for point-source removal from CMB maps that consists of the following two steps:

1. Convolve the map with a band-pass filter such as the one shown in Figure 1.
2. Compute the resulting *rms* fluctuation level σ and interpret all positive fluctuations exceeding $\nu\sigma$ as point sources. Revert to the original map and discard the contaminated pixels.

The convolution kernel W is typically well-localized on the sky, which means that the convolution can be computed directly with a reasonable effort. Alternatively, the convolution can be performed by expanding the sky-map in spherical harmonics, multiplying the expansion coefficients $a_{\ell m}$ by W_ℓ, and transforming back. If necessary, the high-pass filtering can be further accelerated by making a flat-sky approximation locally and performing the convolution using two-dimensional fast Fourier transforms.

We found that this simple method can enhance the detectability of point sources by a substantial factor, which mainly depends on the available signal-to-noise in the unfiltered map. The detectability can of course be further improved by taking an appropriate linear combination of maps at different frequencies (as in Tegmark 1998) before the filtering, to remove *e.g.* the CMB at that stage at the cost of a slight noise increase.

Our filtering procedure also an additional advantage. Since the outlyer removal scheme is more likely to throw away pixels in CMB hot spots than in cold spots, it inevitably introduces a slight bias. With a $5-\sigma$ threshold, a $3-\sigma$ point source will get removed if it resides in a $2-\sigma$ CMB hot spot, whereas it takes a $7-\sigma$ point source to be detected in a corresponding CMB cold spot. By eliminating most of the CMB before the point source removal step, our filtering scheme also eliminates most of this bias.

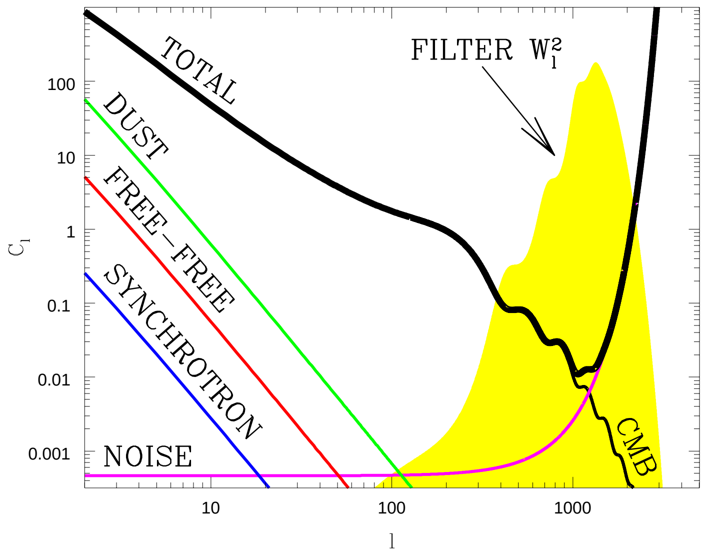

FIG. 2 — Same as Figure 1, but in the Fourier (multipole) domain. W_ℓ^2 is plotted (shaded) together with the total power spectrum C_ℓ^{tot} (heavy line) and the various components that make it up. Everything is for Planck channel 5, *i.e.*, at 100 GHz.

4.1. *Robustness of the results*

How sensitive are our results to the various assumptions that we have made? We found that the Galactic foreground model made only a minimal difference for most Planck channels, where the contribution to C_ℓ^{tot} (and hence to the confusion noise σ) is likely to be dominated by CMB and detector noise for all ℓ. Likewise, we found that changing the cosmological model had a minor effect on σ, with the gain never exceeding 20%. Instead, almost all the uncertainty comes from the assumed source count model (To98), so let us compute this dependence explicitly. If we manage remove all sources brighter than a flux cut S_c, the point source power spectrum in the original (unfiltered) map is (TE96; Tegmark & Villumsen 1997)

$$\frac{1}{c^2}C^{ps} = \int_0^{S_c} n'(S)S^2 dS \approx \left(\frac{\beta-1}{3-\beta}\right) n(S_c)S_c^2, \qquad (11)$$

independent of ℓ, where $n(S)$ is the number of point sources per steradian with a flux exceeding S and $\beta \equiv -d\ln n'/d\ln S$ is the logarithmic slope of the differential source counts $n' \equiv -dn/dS$. This neglects point source clustering, which is generally a good approximation (To98). The rms fluctuations σ_{ps} are given by the familiar expression

$$\sigma_{ps}^2 = C^{ps}\sum_\ell \left(\frac{2\ell+1}{4\pi}\right) B_\ell^2 \approx \frac{1}{4\pi\theta^2}C^{ps}, \qquad (12)$$

where the sum can be accurately approximated by an integral. Above we saw that outlyer removal gave $S_c = \nu\sigma/cB(1) = 2\pi\theta^2\nu\sigma/c$. Substituting this and equation (11) into equation (12), we obtain the useful result

$$\sigma_{ps} \approx \sqrt{\frac{\beta-1}{3-\beta}} N^{1/2}\nu\sigma, \qquad (13)$$

where $N \equiv \pi\theta^2 n(S_c)$ is the number of sources removed per beam area. Since relevant values for β are typically in the range 1.5–2.5 (see references in Tegmark & Villumsen 1997), the first term is of order unity. Table 1 showed that the best attainable σ was typically 3-5 times the rms noise σ_n. Point sources have only a minor impact on a CMB experiment if $\sigma_{ps} \ll \sigma_n$, since their power spectra have the same shape as that for detector noise (apart from the noise blowup up the beam scale). Equation (13) therefore tells us that using the CMB map itself for point source removal is quite adequate as long as $N \ll (4\times5)^{-2} = 0.002$. Conversely, if there are more sources per beam than this rule of thumb indicates, then an external point source template will be needed to reduce the point source contribution to a subdominant level. This criterion thus partitions CMB experiments into two classes: those for which internal cleaning suffices and those which need external point source data.

Since $\sigma_{ps} \propto N^{1/2}$, a useful measure is the *safety margin*, defined as $M \equiv (\sigma_n/\sigma_{ps})^2$. This is the factor by which the number of point sources can be increased before they dominate the noise rms. In Table 1, it is seen to be of order 10 for channels 7 and 8, which means that the models of To98 would need to be off by an order of magnitude for point sources to imperil the Planck mission. For those channels where $M \lesssim 1$ ($\sigma_{ps} \gtrsim \sigma_n$), it will be desirable to use a multi-frequency subtraction scheme or external point source catalogs to further reduce σ_{ps}. Future radio source surveys at CMB frequencies will therefore be extremely valuable to the CMB community.

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$\nu_{\rm eff}$ (GHz)	FWHM (')	$\sigma_{\rm n}$ (mJy)		$\sigma_{\rm Gal}$ (mJy)		$\sigma_{\rm CMB}$ (mJy)		σ (mJy)		Gain	$N(>5\sigma)$ (8 sr)		$\sigma_{\rm ps}$	M (mJy)
		B	A	B	A	B	A	B	A		B	A	A	A
30	33	12	59	80	2	242	66	255	89	2.9	178	867	46	0.06
44	23	19	71	36	1	271	77	274	105	2.6	156	674	33	0.3
70	14	25	78	16	0.5	257	73	262	106	2.5	165	655	19	1.5
100	10	27	79	19	0.5	247	62	252	101	2.5	175	697	13	3.8
100	10.6	13	51	21	0.4	279	48	280	70	4.0	147	1157	12	1.1
143	7.4	11	39	31	0.6	225	31	227	51	4.5	131	1220	6	3.0
217	4.9	14	36	51	1	130	20	141	41	3.4	206	1288	4	14.8
353	4.5	24	50	186	7	69	20	199	54	3.7	205	1712	8	9.0
545	4.5	45	87	609	32	13	6	611	93	6.5	209	4696	24	3.6
857	4.5	36	82	1735	49	0.3	0.06	1735	95	18	293	37612	111	0.1

Table 1 — The rows show how the 10 Planck channels can be used for point source detection before (B) and after (A) filtering. The values of rms confusion noise correspond to detector noise ($\sigma_{\rm n}$), Galactic foregrounds ($\sigma_{\rm Gal}$ is the combined contribution of dust, free-free and synchrotron emission), CMB ($\sigma_{\rm CMB}$), the quadrature sum of all of the above (σ) and unremoved point sources ($\sigma_{\rm ps}$). $\sigma_{\rm CMB}$ is computed from the full power spectrum – using $\sigma_{\rm CMB}= 10^{-5}T_{cmb} \approx 27\mu$K as in *e.g.* To98 gives values about 5 times too low. The gain is the factor by which filtering reduces σ. $N(>5\sigma)$ is the number of detected sources in 8 sr (having $S > S_c$) and M the factor by which the To98 source counts would have to be increased to give $\sigma_{\rm ps}$=$\sigma_{\rm n}$.