SUBMITTED TO APJL APRIL 5 1998; ACCEPTED APRIL 13
Preprint typeset using LaTeX style emulateapj

THE TIME-EVOLUTION OF BIAS

MAX TEGMARK [a,b] AND P. J. E. PEEBLES[c]

[a]Institute for Advanced Study, Princeton, NJ 08540; max@ias.edu
[b]Hubble Fellow
[c] Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544; pjep@pupgg.princeton.edu

Submitted to ApJL April 5 1998; accepted April 13

ABSTRACT

We study the evolution of the bias factor b and the mass-galaxy correlation coefficient r in a simple analytic model for galaxy formation and the gravitational growth of clustering. The model shows that b and r can be strongly time-dependent, but tend to approach unity even if galaxy formation never ends as the gravitational growth of clustering debiases the older galaxies. The presence of random fluctuations in the sites of galaxy formation relative to the mass distribution can cause large and rapidly falling bias values at high redshift.

Subject headings: galaxies: statistics — large-scale structure of universe

1. INTRODUCTION

The relative distribution of galaxies and mass is of increasing concern in cosmology. Constraints on cosmological parameters from galaxy surveys are only as accurate as our understanding of bias. Furthermore, if the density parameter in mass that is capable of clustering were shown to be $\Omega = 0.25 \pm 0.15$ then most dynamical analyses of the motions of galaxies would be consistent with the assumption that galaxies trace mass, and the challenge would be to explain why simulations of the adiabatic cold dark matter (CDM) model for structure formation typically indicate galaxies are more strongly clustered than mass (Jenkins *et al.* 1998). If $\Omega = 1$, galaxies do not trace mass and it would be puzzling that the mass autocorrelation function in CDM simulations is a much poorer approximation to a power law on scales 10 kpc $\lesssim hr \lesssim$ 10 Mpc than is the galaxy two-point correlation function, and that the mass function shows a much more pronounced evolution of shape back to $z \sim 1$ (Jenkins *et al.* 1998). Major surveys in progress of galaxies and the cosmic microwave background (the CMB) will advance our understanding of such issues. It may be useful to supplement these observations with analytic illustrations of how biasing can evolve as mass clustering grows and gravity draws together the galaxies with the mass. The purpose of this *Letter* is to present a simple analytic model for the dynamical evolution of the relative distribution of galaxies and mass.

We assume that at formation a galaxy may be assigned a near permanent observational tag. The far infrared luminosity would not do, for IRAS galaxies avoid dense regions, an example of biasing due to environment (*e.g.*, Strauss & Willick 1995). The spheroid luminosity may be an adequate tag, for the spheroid star populations in more luminous galaxies are thought to be old and slowly evolving. We let $\rho_l(\mathbf{r})$ be the density of this luminous matter and $\rho(\mathbf{r})$ be the total mass density, both smoothed on some appropriate scale R. The corresponding density fluctuations are $\delta \equiv \rho/\langle\rho\rangle - 1$ and $\delta_l \equiv \rho_l/\langle\rho_l\rangle - 1$. In a commonly used model $\delta_l = b\delta$, where the constant b is the bias factor. We adopt the more general and possibly more realistic statistical representation of Dekel & Lahav (1998; Dekel 1997 §5.5; Lahav 1996 §3.1) where δ and δ_l are treated as stationary (translationally invariant) random processes that may be grouped in the two-dimensional vector

$$\mathbf{x} \equiv \begin{pmatrix} \delta \\ \delta_l \end{pmatrix}. \tag{1}$$

The mean $\langle\mathbf{x}\rangle$ vanishes by definition. We write the covariance matrix as

$$\mathbf{C} \equiv \langle\mathbf{x}\mathbf{x}^t\rangle = \sigma^2 \begin{pmatrix} 1 & br \\ br & b^2 \end{pmatrix}, \tag{2}$$

where $\sigma \equiv C_{11}^{1/2}$ is the rms mass fluctuation on the scale R, $b \equiv (C_{22}/C_{11})^{1/2}$ is the bias factor (the ratio of luminous and total fluctuations), and $r \equiv C_{12}/(C_{11}C_{22})^{1/2}$ is the dimensionless correlation coefficient of the distributions of mass and galaxies. If the smoothing scale R is large enough that $\mathbf{x}$ can be modeled as a bivariate Gaussian random variable, then $\mathbf{C}$ contains all the statistical information about $\mathbf{x}$. In principle this representation is not affected by galaxy merging, if the process does not add much to the star population that tags galaxies. In practice many of the surveys in progress will use galaxy counts, but it is thought that there has not been substantial merging of $L \gtrsim L_*$ galaxies since redshift $z = 1$. Pen (1998) has shown that the three quantities σ, b and r can be measured from redshift space distortions (Kaiser 1987; reviewed by Hamilton 1997). High redshift surveys in progress (see *e.g.* Giavalisco *et al.* 1998; Yee *et al.* 1998 and the review in Moscardini *et al.* 1998) open the possibility of measuring the time evolution of b and r. Thus it seems timely to explore physical models for the evolution of these second moment measures of biasing.

For studying the scale-dependence of bias, it is useful to work with $\widehat{\mathbf{x}}$, the Fourier transform of $\mathbf{x}$, which satisfies $\langle\widehat{\mathbf{x}}(\mathbf{k})\widehat{\mathbf{x}}(\mathbf{k}')^\dagger\rangle = (2\pi)^3\delta^D(\mathbf{k}-\mathbf{k}')\widehat{\mathbf{C}}(\mathbf{k})$,

$$\widehat{\mathbf{C}}(\mathbf{k}) \equiv \begin{pmatrix} P(\mathbf{k}) & P_\times(\mathbf{k}) \\ P_\times(\mathbf{k}) & P_l(\mathbf{k}) \end{pmatrix} = P(\mathbf{k}) \begin{pmatrix} 1 & b'r' \\ b'r' & b'^2 \end{pmatrix}. \tag{3}$$

Here P is the power spectrum of the mass distribution, P_l is the power spectrum of the light that traces the galaxies, and $P_\times$ is the cross spectrum. The last expression defines the analogs of the bias functions in equation (2). We have entered vector arguments to take account of application in redshift space in fields of small angular width. The following analysis applies equally well to b and r as to b' and r'.

Most previous analyses of biasing models have focused on the static and local or non-local relation between δ_l and δ as galaxies form. Recent reviews are given by Mann *et al.* (1998), Croft etal (1998) and Scherrer & Weinberg (1998); the latter appears to be the first to make static model predictions for r as well as b. But these measures are functions of time. Thus even if galaxies initially were uncorrelated with the mass ($r = 0$), they would gradually become correlated as gravity draws them towards overdense regions, and one might expect this process to drive b and r toward unity. Fry (1996) has demonstrated this explicitly for the special case $r = 1$, and similar conclusions have been found in numerical simulations of dark matter halo clustering (*e.g.*, Mo & White 1996; Matarrese *et al.* 1997; Bagla 1998; Wechsler *et al.* 1998).

We limit our discussion to the linear perturbation theory regime $|\delta| \ll 1$, that is, to large scales R. In §2 we derive a general expression for the time-evolution of $\mathbf{C}$ that applies once galaxy formation has stopped. We generalize to ongoing galaxy formation in §3, and present conclusions in §4.

2. AFTER THE GALAXY FORMATION EPOCH

We assume galaxy formation converts part of the mass into a near permanent luminous form without affecting its position or velocity on large scales, and that this luminous form behaves as test particles that are fair tracers of the large-scale velocity field (but not necessarily the mass density). Since galaxies and mass are assumed to have the same bulk peculiar velocity $\mathbf{v}(\mathbf{r})$, the contrasts δ_l and δ satisfy the same linear continuity equation, $\dot{\delta}_l \approx \dot{\delta} \approx -\nabla\cdot\mathbf{v}$, in the absence of galaxy formation. This gives

$$\mathbf{x}_0 = \mathbf{M}\mathbf{x}, \quad \text{where} \quad \mathbf{M} \equiv \begin{pmatrix} D^{-1} & 0 \\ D^{-1}-1 & 1 \end{pmatrix}. \tag{4}$$

The subscript indicates the present value, and D is the growth factor of the mass density contrast in linear perturbation theory (see *e.g.* Peebles 1980) normalized to the present value $D(a_0) = 1$. The covariance matrix therefore evolves to

$$\mathbf{C}_0 = \mathbf{M}\mathbf{C}\mathbf{M}^t. \tag{5}$$

Equations (2) to (5) give

$$\begin{aligned} \sigma_0 &= \sigma/D, & (6)\\ b_0 &= [(1-D)^2 + 2D(1-D)br + D^2b^2]^{1/2}, & (7)\\ r_0 &= [(1-D) + Dbr]/b_0, & (8) \end{aligned}$$

The first equation simply states that the total mass fluctuations have grown as $D(t)$. The second two equations show that *the situation tends to grow simpler*, b and r approaching unity regardless of their initial values. This is illustrated in Figure 1. Inverting equations (6) to (8) gives

$$\begin{aligned} \sigma &= \sigma_0 D, & (9)\\ b &= [(1-D)^2 - 2(1-D)b_0r_0 + b_0^2]^{1/2}/D, & (10)\\ r &= [b_0r_0 - (1-D)]/Db. & (11) \end{aligned}$$

This tells us what σ, b and r must have been in the past to produce the present values. In an Einstein-de Sitter universe ($\Omega_m = 1$ and $\Omega_\Lambda = 0$, as in the standard SCDM cold dark matter model), the linear growth factor is $D = a/a_0 = (1+z)^{-1}$ and these equations are

$$\begin{aligned} \sigma &= (1+z)^{-1}\sigma_0, & (12)\\ b &= [z^2 - 2z(1+z)b_0r_0 + (1+z)^2b_0^2]^{1/2}, & (13)\\ r &= [(1+z)b_0r_0 - z]/b. & (14) \end{aligned}$$

For reference, these SCDM redshifts z are given at the top of Figure 1.

The value $r = 1$ is a fixed point: if mass and galaxies are perfectly correlated at one time this remains true for all time. For $r = 1$, equation (7) is

$$(b_0 - 1) = (b-1)D, \tag{15}$$

which for SCDM gives $b = b_0 + (b_0-1)z$, the special case derived by Fry (1996) and Mo *et al.* (1997). This is the top curve in each set in Figure 1A. One also sees that if $r \sim 0$, b tends to decrease even if there is no initial bias. The correlation r grows monotonically with time in all cases. The larger b, the slower the approach of r to unity.

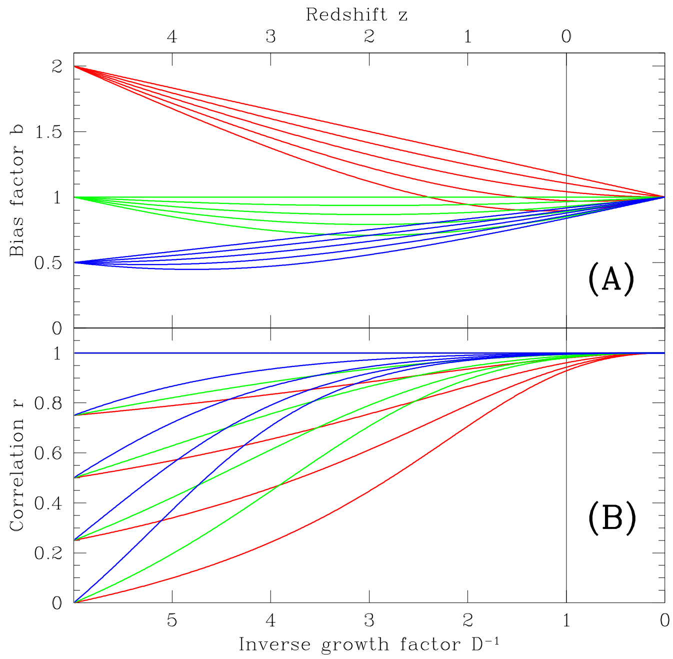

FIG. 1 — The evolution of bias (A) and correlation (B) is shown for 15 models that at Einstein-de Sitter redshift $z = 5$ have bias $b = 0.5$, 1, and 2 and correlation coefficients $r = 0$, 0.25, 0.5, 0.75, and 1. In the top panel, r increases upward in each quintuplet of lines. In the bottom panel, b_0 increases downward in each triplet of lines. The present epoch is at $D^{-1} = 1$, and the evolution is extrapolated to a large future increase in the density contrast, $D^{-1} \to 0$. The redshift scale at the top of the figure assumes the Einstein-de Sitter model.

3. ONGOING GALAXY FORMATION

We consider now the more general case where galaxies form over a substantial range of redshift. Here there is a competition between gravitational instability, which pushes b and r toward unity, and the formation of new galaxies, which can be biased or poorly correlated with the mass distribution. We model the time evolution of the density of the galaxies, as measured by the luminosity tracer ρ_l, as

$$\dot{\rho}_l + \rho_l \nabla\cdot\mathbf{v} = g(t)\left[1 + b_*\delta + \delta_\perp\right], \qquad (16)$$

in comoving coordinates. The galaxy formation rate per unit volume is $g(t)$. The dimensionless factor in brackets models the sum of galaxy formation determined by the local mass density and a random component $\delta_\perp(\mathbf{r},t)$. The latter has zero mean ($\langle\delta_\perp\rangle = 0$) and is uncorrelated with the mass density ($\langle\delta\delta_\perp\rangle = 0$). The deterministic part is represented as the first two terms in a Taylor series expansion of the galaxy formation rate as a function of the local mass density, as in Coles (1993) and Fry & Gaztañaga (1993). The time-dependent parameter b_* in this expansion is the "bias at birth".

As in the previous section, the streaming velocity of galaxies and mass is related to the time-evolution of the mass contrast, $-\nabla\cdot\mathbf{v} = \dot{\delta} = \dot{D}\delta_0$. The space average of equation (16) is to leading order $\langle\dot{\rho}_l\rangle = g(t)$, so the mean galaxy density is

$$\langle\rho_l(t)\rangle = G(t) \equiv \int_0^t g(t')dt'. \qquad (17)$$

Thus we can rewrite equation (16) as

$$\dot{\rho}_l = g + (b_* D g + \dot{D}G)\delta_0 + g\delta_\perp, \qquad (18)$$

and we integrate this to obtain ρ_l. For any time-dependent quantity $f(t)$ we define the time average weighted by the galaxy formation rate g by

$$\langle f\rangle_t \equiv \frac{1}{G(t)}\int_0^t f(t')g(t')dt' = \frac{1}{G(t)}\int_0^{G(t)} f dG'. \qquad (19)$$

With this notation and the relation $\dot{D}G = (DG)^{\cdot} - Dg$, equation (18) gives the galaxy contrast

$$\delta_l \equiv \frac{\rho_l}{\langle\rho_l\rangle} - 1 = c\delta + \langle\delta_\perp\rangle_t, \qquad (20)$$

where

$$c \equiv 1 + \frac{\langle(b_* - 1)D\rangle_t}{D}. \qquad (21)$$

Then the covariance matrix is

$$\mathbf{C} = \langle\mathbf{x}\mathbf{x}^t\rangle = \sigma^2\begin{pmatrix} 1 & c \\ c & c^2 + s^2/\sigma^2\end{pmatrix}, \qquad (22)$$

where

$$s^2 \equiv \frac{1}{G^2}\int_0^G\int_0^G \langle\delta_\perp(G')\delta_\perp(G'')\rangle dG' dG'', \qquad (23)$$

in the notation of the last term in equation (19). Combining equation (2) with equation (22) yields

$$b = [c^2 + s^2/\sigma^2]^{1/2}, \qquad r = c/b. \qquad (24)$$

Let us first consider some simple cases. If there is no random contribution to galaxy formation, $\delta_\perp = 0$, then $s^2 = 0$, $r = 1$, and $b = c$. If $r = 1$ and the bias at birth, b_*, is time-independent, then equation (21) reduces to $(b_0 - 1) = (b_* - 1)\langle D\rangle_t$. Here the deviation of the bias from unity is suppressed by the average growth factor between the galaxy formation epoch and today. This is just equation (15) averaged over the galaxy formation history. In the the Einstein-de Sitter model this bias suppression factor $\langle D\rangle_t$ is $\langle(1+z)^{-1}\rangle_t$.

Table 1 – Resulting b_0 and r_0 for various models, for a constant galaxy formation rate $g(z)$ between redshifts z_{on} and z_{off}.

Model	Ω_0	Λ_0	z_{on}	z_{off}	b_*	s_*	N	b_0	r_0
M1	1	0	5	5	2	0	1	1.17	1
M2	0.3	0.7	5	5	2	0	1	1.21	1
M3	0.3	0	5	5	2	0	1	1.30	1
M4	1	0	5	0	2	0	1	1.36	1
M5	1	0	1	0	2	0	1	1.69	1
M6	1	0	3	1	2	0	1	1.35	1
M7	1	0	5	2	2	0	1	1.23	1
M8	1	0	7	3	2	0	1	1.17	1
M9	1	0	5	2	0.5	0	1	0.88	1
M10	1	0	5	2	1	0	1	1	1
M11	1	0	5	2	1	0.3	1	1.08	.93
M12	1	0	5	2	1	0.3	10	1.01	.99
M13	1	0	5	2	2	0.3	1	1.29	.95
M14	1	0	5	2	2	0.3	10	1.24	.99

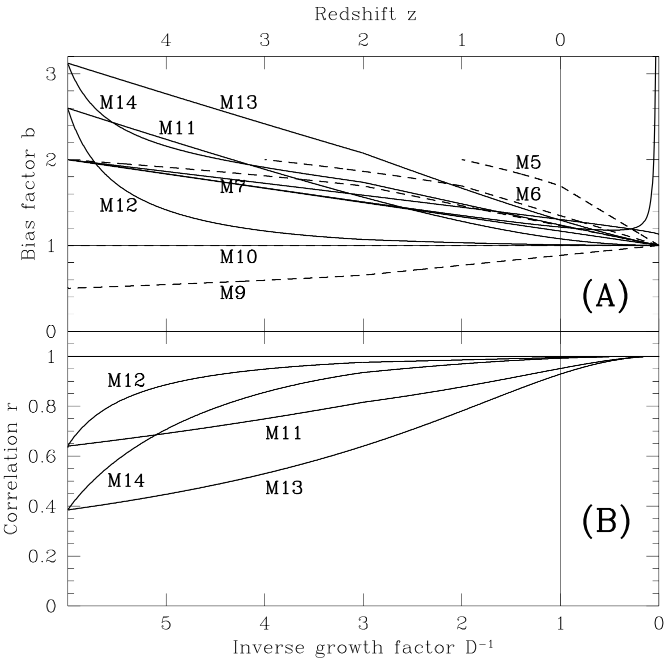

FIG. 2 — The time-evolution of bias (A) and correlation (B) for some of the models in Table 1.

The randomness contribution s^2 to the correlation r depends on $\langle\delta_\perp(t')\delta_\perp(t'')\rangle$, which is a measure of the memory of fluctuations $\delta_\perp$ at the same comoving position at different times. If memory were complete, $\delta_\perp$ a function of position alone, then $s = \langle\delta_\perp^2\rangle_t^{1/2}$. In the limit of a short coherence time, $\langle\delta_\perp(G')\delta_\perp(G'')\rangle \propto \delta(G'-G'')$, equation (23) gives $s \propto G^{-1/2}$, just as the average of N independent fluctuations is proportional to $N^{-1/2}$. The simple two-parameter model

$$s = \frac{s_* \sigma_0}{\sqrt{1 + (N-1)\frac{G}{G(a_0)}}} \qquad (25)$$

incorporates both of these extremes as special cases ($N = 1$ and $N = \infty$, respectively), and since this aspect of the galaxy formation process is still so poorly understood a more complicated model for s does not yet seem warranted. The parameter N can be interpreted as the effective number of uncorrelated galaxy formation epochs. The current rms mass contrast is σ_0, and the other model parameter is the relative normalization s_*.

Examples of these relations are listed in Table 1 and plotted in Figure 2. The growth factors D for the low density models are computed as in Carroll *et al.* (1992). These models show that the biasing parameters r and b can vary quite rapidly near the start of galaxy formation, and that r and b tend to unity even if galaxy formation never ends, as in models 4 and 5, because the growth of gravitational clustering debiases the growing number of older galaxies. After galaxy formation terminates b and r approach unity more rapidly, evolving as in Figure 1. A random component $\delta_\perp$ produces a large early value of b, but b and r approach unity quite rapidly if $N \gg 1$, as the fluctuations from large numbers of random events average down to insignificant levels.

4. CONCLUSIONS

We have computed the evolution of the bias factor b and the mass-galaxy correlation coefficient r in a simple analytic model represented by a galaxy formation history $g(z)$, the parameters in a representation of biased galaxy formation (b_*, s_*, and N), and the parameters for the cosmological model (Ω and Λ). Galaxy formation could be affected by explosions, as from quasars, at positions unrelated to the local mass density (*e.g.*, Dekel & Rees 1987; Babul & White 1991). This effect is represented by the term $\delta_\perp$. The coherence time of $\delta_\perp$, as measured by N, would depend on how long individual quasars last and how long the intergalactic medium "remembers" their effects. The complications of luminosity evolution and galaxy mergers are incorporated into our formalism by defining $g(z)$ to be the formation history of the matter that is luminous at the redshift z' at which we wish to evaluate b and r.

If galaxies were assembled at $z \lesssim 1$ there could be a large and observationally significant evolution of the biasing functions b and r at low redshift, as illustrated by the steep initial slopes in Figure 2. On the other hand, if galaxy positions were assigned at relatively high redshift the effect of biasing at birth would be strongly suppressed. Decreasing the mass density Ω decreases this effect, because the growth of density perturbations at low redshifts is slower, but this likely is at least partially canceled by shift of the formation epoch to larger redshifts. In fact, if g, b_*, and $\delta_\perp$ were functions of D alone, debiasing would be independent of the cosmology.

The new galaxy redshift surveys, including the two degree field (2dF) survey and the Sloan Digital Sky Survey, have the potential to measure key cosmological parameters with great accuracy, both alone (Tegmark 1997; Goldberg & Strauss 1998), and when combined with cosmic microwave background experiments (Hu *et al.* 1997). To achieve this, however, biasing and its evolution must be understood to a comparable accuracy. Our formalism deals with the second moments of the mass and galaxy distributions in a universe that is statistically homogeneous (eq. [2]). The trend to debiasing of the functions b and r in this representation appears to be a generic feature of the gravitational instability scenario for structure formation, a prediction that may be observationally tested in the near future as galaxy redshift surveys improve.

We thank Ben Bromley and Ue-Li Pen for useful discussions. Support for this work was provided by NASA though grant NAG5-6034 and a Hubble Fellowship, HF-01084.01-96A, awarded by STScI, which is operated by AURA, Inc. under NASA contract NAS5-26555, and at Princeton University by the National Science Foundation.

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